IT may be proper to acquaint the Reader, that we have in this Edition added, in the Margin, References to the *Engliſh* Tranſlation of the *Principia* lately publiſhed, directing to the ſeveral Pages in that Tranſlation, where the Things here treated of are proved: And that the Aſterisk in the Margin is deſigned to ſhew how far the Diſcourſe in the *Principia* is the ſame with what is found in this Treatiſe.

A
TREATISE
OF THE
SYSTEM
OF THE
WORLD.

BY
Sir *ISAAC NEWTON.*

Tranſlated into ENGLISH.

The THIRD EDITION.

LONDON:
Printed for B. MOTTE and C. BATHURST,
at the *Middle Temple-Gate*, in *Fleet-ſtreet.*

MDCCXXXVII.

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30 OCT 64

1608/2139

From Sir ISAAC NEWTON'*s Mathematical Principles*, Lib. III. Introd.

IN the preceding books I have laid down the principles of philoſophy; principles, not philoſophical, but mathematical; ſuch, to wit, as we may build our reaſonings upon in philoſophical enquiries. Theſe principles are the laws and conditions of certain motions, and powers or forces, which chiefly have reſpect to philoſophy. But leſt they ſhould have appeared of themſelves dry and barren, I have illuſtrated them here and there with ſome philoſophical ſcholiums, giving an account of ſuch things as are of more general nature, and which philoſophy ſeems chiefly to be founded on; ſuch as the denſity and the reſiſtance of bodies, ſpaces void of all bodies, and the motion of light and ſounds. It remains, that from the ſame principles I now demonſtrate the frame of the SYSTEM of the WORLD. Upon this ſubject, I had indeed compoſed the third book in a popular method, that it might be read by many: but afterwards conſidering that ſuch as had not ſufficiently entered into the principles, could not eaſily diſcern the ſtrength of the conſequences, nor lay aſide the prejudices to which they had been many years accuſtomed; therefore to prevent the diſputes which might be raiſed upon ſuch accounts, I choſe to reduce the ſubſtance of that book into the form of propoſitions (in the mathematical way) which ſhould be read by thoſe only, who had firſt made themſelves maſters of the principles eſtabliſhed in the preceding books.

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OF THE SYSTEM OF THE WORLD.

That the matter of the Heavens is fluid.

IT was the ancient opinion of not a few in the earlieſt ages of philoſophy, That the fixed Stars ſtood immoveable in the higheſt parts of the world; that under the Fixed Stars the Planets were carried about the Sun; that the Earth, as one of the Planets, deſcribed an annual courſe about the Sun, while by a diurnal motion it was in the mean time revolved about its own axe; and that the Sun, as the common fire which ſerved to warm the whole, was fixed in the center of the Univerſe.

Archimedes *in Arenario*. Ariſtot. *Lib.* 2. *de cœlo.* Plutarch, *Lib.* 3. *de placitis Philoſ.* & *in* Numa.

This was the philoſophy taught of old by *Philolaus*, *Ariſtarchus* of *Samos*, *Plato* in his riper years, and the whole ſect of the *Pythagoreans*. And this was the judgment of *Anaximander*, more ancient than any of them, and of that wiſe king of the *Romans*, *Numa Pompilius*; who, as a ſymbol of the figure of the World with the Sun in the center, erected a temple in honour of *Veſta*, of a round form, and ordained perpetual fire to be kept in the middle of it.

The *Egyptians* were early obſervers of the heavens. And from them probably this philoſophy was ſpread abroad among other nations. For from them it was, and the nations about them, that the *Greeks*, a people of themſelves more addicted to the ſtudy of philology than of nature, derived their firſt, as well as ſoundeſt, notions of philoſophy. And in the veſtal ceremonies we may yet trace the ancient ſpirit of the *Egyptians*. For it was their way to deliver their myſteries, that is, their philoſophy of things above the vulgar way of thinking, under the veil of religious rites and hieroglyphick ſymbols.

It is not to be denied but that *Anaxagoras*, *Democritus* and others did now and then ſtart up, who would have it that the Earth poſſeſſed the center of

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the World, and that the Stars of all ſorts were revolved towards the weſt, about the Earth quieſcent in the center; ſome at a ſwifter, others at a ſlower rate.

However, it was agreed on both ſides, that the motions of the celeſtial bodies were performed in ſpaces altogether free, and void of reſiſtance. The whim of ſolid orbs was of a later date, introduced by *Eudoxus*, *Calippus* and *Ariſtotle*; when the ancient philoſophy began to decline, and to give place to the new prevailing fictions of the *Greeks*.

But above all things, the phænomena of Comets can by no means conſiſt with the notion of ſolid orbs. The *Chaldeans*, the moſt learned aſtronomers of their time, looked upon the Comets (which of ancient times before had been numbered among the celeſtial bodies) as a particular ſort of planets, which, deſcribing very eccentric orbits, preſented themſelves to our view only by turns, *viz.* once in a revolution, when they deſcended into the lower parts of their orbits.

And as it was the unavoidable conſequence of the hypotheſis of ſolid orbs, while it prevailed, that the Comets ſhould be thruſt down below the Moon; ſo no ſooner had the late obſervations of aſtronomers reſtored the Comets to their ancient places in the higher heavens, but theſe celeſtial ſpaces were at once cleared of the incumbrance of ſolid orbs, which by theſe obſervations were broke into pieces and diſcarded for ever.

The principle of circular motion in free ſpaces.

WHENCE it was that the Planets came to be retained within any certain bounds in theſe free ſpaces, and to be drawn off from the rectilinear courſes, which, left to themſelves, they ſhould have purſued, into regular revolutions in curvilinear orbits, are queſtions which we do not know how the ancients explained. And probably it was to give ſome ſort of ſatisfaction to this difficulty, that ſolid orbs were introduced.

The later philoſophers pretend to account for it, either by the action of certain vortices, as *Kepler* and *Des Cartes*; or by ſome other principle of impulſe or attraction, as *Borelli*, *Hook*, and others of our nation. For, from the laws of motion, it is moſt certain that theſe effects muſt proceed from the action of ſome force or other.

But our purpoſe is only to trace out the quantity and properties of this force from the phænomena, and to apply what we diſcover in ſome ſimple caſes, as principles, by which, in a mathematical way,

P. 262. *Vol.* I. Princip.

way, we may eſtimate the effects thereof in more involved caſes. For it would be endleſs and impoſſible to bring every particular to direct and immediate obſervation.

We ſaid, *in a mathematical way*, to avoid all queſtions about the nature or quality of this force, which we would not be underſtood to determine by any hypotheſis; and therefore call it by the general name of a centripetal force, as it is a force which is directed towards ſome center; and as it regards more particularly a body in that center, we call it circum-ſolar, circum-terreſtrial, circum-jovial, and in like manner in reſpect of other central bodies.

The effects of centripetal forces.

THAT by means of centripetal forces, the Planets may be retained in certain orbits, we may eaſily underſtand, if we conſider the motions of projectiles. For a ſtone projected is by the preſſure of its own weight forced out of the rectilinear path, which by the projection alone it ſhould have purſued, and made to deſcribe a curve line in the air; and through that crooked way is at laſt brought down to the ground. And the greater the velocity is with which it is projected, the farther it goes before it falls to the Earth. We may therefore

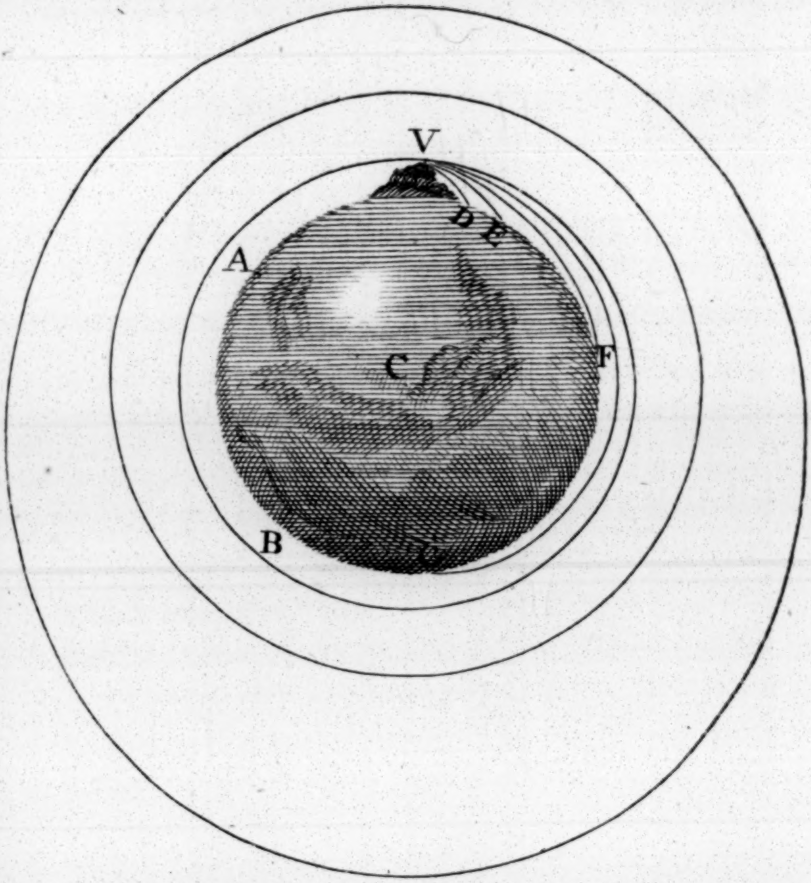
P. 4, 5, 6. *Vol.* I.

fore ſuppoſe the velocity to be ſo encreaſed, that it would deſcribe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the Earth, till at laſt exceeding the limits of the Earth, it ſhould paſs quite by without touching it.

Let AFB repreſent the ſurface of the Earth, C its center, VD, VE, VF, the curve lines which a body would deſcribe, if projected in an horizontal direction from the top of an high mountain, ſucceſſively with more and more velocity. And, becauſe the celeſtial motions are ſcarcely retarded by the little or no reſiſtance of the ſpaces in which they are performed; to keep up the parity of caſes, let us ſuppoſe either that there is no air about the Earth, or at leaſt that it is endowed with little or no power of reſiſting. And for the ſame reaſon that the body projected with a leſs velocity, deſcribes the leſſer arc VD, and with a greater velocity, the greater arc VE, and augmenting the velocity, it goes farther and farther to F and G; if the velocity was ſtill more and more augmented, it would reach at laſt quite beyond the circumference of the Earth, and return to the mountain from which it was projected.

*Vid.Vol.*II. Princip. *p.* 230.

And ſince the area's, which by this motion it deſcribes by a radius drawn



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to the center of the Earth, are (by *Prop.* 1. *Book* 1. *Princip. Math.*) proportional to the times in which they are deſcribed; its velocity, when it returns to the mountain, will be no leſs than it was at firſt; and retaining the ſame velocity, it will deſcribe the ſame curve over and over, by the ſame law.

But if we now imagine bodies to be projected in the directions of lines parallel to the horizon from greater heights, as of 5, 10, 100, 1000 or more miles, or rather as many ſemi-diameters of the Earth; thoſe bodies, according to their different velocity, and the different force of gravity in different hights, will deſcribe arcs either concentric with the Earth, or variouſly excentric, and go on revolving through the heavens in thoſe trajectories, juſt as the Planets do in their orbs.

The certainty of the argument.

As when a ſtone is projected obliquely, that is, any way but in the perpendicular direction, the perpetual deflection thereof towards the Earth from the right line in which it was projected, is a proof of its gravitation to the Earth, no leſs certain than its direct deſcent when only ſuffered to fall freely from reſt; ſo the deviation of bodies, moving in free ſpaces, from rectilinear paths, and perpetual deflexion therefrom towards any place, is a ſure indication of the exiſtence of ſome force, which from all quarters impells thoſe bodies towards that place.

And as from the ſuppoſed exiſtence of gravity, it neceſſarily follows that all bodies about the Earth muſt preſs downwards, and therefore muſt either deſcend directly to the Earth, if they are let fall from reſt, or at leaſt perpetually deviate from right lines towards the Earth, if they are projected obliquely; ſo from the ſuppoſed exiſtence of a force directed to any center, it will follow by the like neceſſity, that all bodies, upon which this force acts, muſt either deſcend directly to that center, or at leaſt deviate perpetually towards it from right lines, if otherwiſe they ſhould have moved obliquely in theſe right lines.

And how from the motions given we may infer the forces, or from the forces given we may determine the motions, is ſhewn in the two firſt Books of our *Principles of Philoſophy*.

What follows from the ſuppoſed diurnal motion of the ſtars.

If the Earth is ſuppoſed to ſtand ſtill, and the fixed Stars to be revolved in free ſpaces, in the ſpace of 24 hours, it is certain the forces by which the fixed Stars are retained in their orbs, are not directed to the Earth, but to the centers of the ſeveral orbs, that is, of the ſeveral parallel circles, which the fixed Stars, declining to one ſide and the other from the Equator, deſcribe daily. Alſo that by radii drawn to the centers of thoſe orbs, the fixed Stars deſcribe area's exactly proportional to the times of deſcription. Then becauſe the periodic times are equal, it follows that the centripetal forces are as the radii of the ſeveral orbs; and that they will perpetually revolve in the ſame orbs. And the like conſequences may be drawn from the ſuppoſed diurnal motion of the planets.

By Cor. 3. *Prop.* 4. *Book* 1.

The incongruous conſequences of this ſuppoſition.

THAT forces ſhould be directed to no Body on which they phyſically depend, but to innumerable imaginary points in the axe of the Earth is an hypotheſis too incongruous. 'Tis more incongruous ſtill that thoſe forces ſhould increaſe exactly in proportion of the diſtances from this axe. For this is an indication of an increaſe to immenſity, or rather to infinity; whereas the forces of natural things commonly decreaſe in receding from the fountain from which they flow. But what is yet more abſurd, neither are the area's, deſcribed by the ſame Star, proportional to the times, nor are its revolutions performed in the ſame orb. For as the Star recedes from the neighbouring pole, both area's and orb increaſe; and from the increaſe of the area, it is demonſtrated that the forces are not directed to the axe of the Earth. *Cor. 1. Prop. 2.* And this difficulty ariſes from the twofold motion that is obſerved in the fixed Stars, one diurnal round the axe of the Earth, the other exceeding ſlow, round the axe of the ecliptic. And the explication thereof requires a compoſition of forces ſo perplexed and ſo variable, that it is hardly to be reconciled with any phyſical theory.

That there is a centripetal force really directed to the center of every planet.

THAT there are centripetal forces actually directed to the bodies of the Sun, of the Earth, and other Planets, thus I infer.

The Moon revolves about our Earth, and by radii drawn to its center, *P. 212.* deſcribes area's nearly proportional to the times in which they are deſcribed; as is evident from its velocity compared with its apparent diameter. For its motion is ſlower, when its diameter is leſs (and therefore its diſtance greater) and its motion is ſwifter when its diameter is greater.

The revolutions of the ſatellits of Jupiter about that planet are more regular. For they deſcribe circles concentric with Jupiter by equable motions, as exactly as our ſenſes can diſtinguiſh. P. 206.

And ſo the ſatellits of Saturn are revolved about this planet with motions nearly circular and equable, ſcarcely diſturbed by any eccentricity hitherto obſerved. P. 208.

That Venus and Mercury are revolved about the Sun, is demonſtrable from their Moon-like appearances. When they ſhine with a full face, they are in thoſe parts of their orbs which in reſpect of the Earth lie beyond the Sun; when they appear half full, they are in thoſe parts which lie over againſt the Sun; when horned, in thoſe parts which lie between the Earth and the Sun, and ſometimes they paſs over the Sun's diſc, when directly interpoſed between the Earth and the Sun. P. 209.

And Venus, with a motion almoſt uniform, deſcribes an orb nearly circular and concentric with the Sun.

But Mercury, with a more eccentric motion, makes remarkable approaches to the Sun, and goes off again by turns; but it is always ſwifter as it is near to the Sun, and therefore by a radius drawn to the Sun, ſtill deſcribes area's proportional to the times.

Laſtly, that the Earth deſcribes about the Sun, or the Sun about the Earth, by a radius from the one to the other, areas exactly proportional to the times, is demonſtrable from the apparent diameter of the Sun compared with its apparent motion.

Theſe are aſtronomical experiments, from which it follows, by *Prop.* 1, 2, 3, in the firſt Book of our *Principles*, and their *Corollaries*, that there are centripetal forces, actually directed (either accurately or without conſiderable errour) to the centers of the Earth, of Jupiter, of Saturn, and of the Sun. In Mercury, Venus, Mars, and the leſſer planets, where experiments are wanting, the arguments from analogy muſt be allowed in their place. *P.* 213, 214, 215, 219.

P. 213, 214, 219.

That thoſe centripetal forces decreaſe in the duplicate proportion of the diſtances from the center of every planet.

THAT thoſe forces decreaſe in the duplicate proportion of the diſtances from the center of every planet, appears by *Cor.* 6. *Prop.* 4. *Book* 1. For the periodic times of the ſatellits of Jupiter are, one to another, in the ſeſquiplicate proportion of their diſtances from the center of this planet. *P.* 206, 207.

This proportion has been long ago obſerved in thoſe ſatellits. And Mr. *Flamſteed,*

ſteed, who had often meaſured their diſtances from Jupiter by the micrometer, and by the eclipſes of the ſatellits, wrote to me, that it holds to all the accuracy that poſſibly can be diſcerned by our ſenſes. And he ſent me the dimenſions of their orbits taken by the micrometer, and reduced to the mean diſtance of Jupiter from the Earth, or from the Sun, together with the times of their revolutions, as follows:

	The greateſt elongation of the ſatellits from the center of *Jupiter* as ſeen from the Sun.				The periodic times of their revolutions.			
	'	''		''	*d.*	*h.*	'	''
1ſt.	1	48	or	108	1	18	28	36
2d.	3	01	or	181	3	13	17	54
3d.	4	46	or	286	7	03	59	36
4th.	8	13½	or	493½	16	18	5	13

Whence the ſeſquiplicate proportion may be eaſily ſeen. For example, the $16^{d}\ 18^{h}\ 05'\ 13''$ is to the time $1^{d}.\ 18^{h}\ 28'\ 36''$ as $493\frac{1}{2}'' \times \sqrt{493\frac{1}{2}''}$ to $108'' \times \sqrt{108''}$, neglecting thoſe ſmall fractions which, in obſerving cannot be certainly determined.

Before the invention of the micrometer, the ſame diſtances were determined in ſemi-diameters of Jupiter, thus:

Di-

Diſtance of the	1ſt.	2d.	3d.	4th.
By *Galilæo*	6	10	16	28
Simon Marius	6	10	16	26
Caſſini	5	8	13	23
Borelli more exactly.	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$

After the invention of the micrometer.

By *Townley*	5,51	8,78	13,47	24,72
Flamſteed	5,31	8,85	13,98	24,23
More accurately by the eclipſes	5,578	8,876	14,159	24,903

And the periodic times of thoſe ſatellits, by the obſervations of Mr. *Flamſteed*, are $1^{d}\ 18^{h}\ 28'\ 36''$ | $3^{d}\ 17^{h}\ 17'\ 54''$ | $7^{d}\ 3^{h}\ 59'\ 36''$ | $16^{d}\ 18^{h}\ 5'\ 13''$ as above.

And the diſtances thence computed are 5,578 | 8,878 | 14,168 | 24,968 | accurately agreeing with the diſtances by obſervation.

[*] P. 208, 209.

Caſſini aſſures us that the ſame proportion is obſerved in the circum-ſaturnal planets. But a longer courſe of obſervations is required before we can have a certain and accurate theory of thoſe planets.

In the circum-ſolar planets, Mercury and Venus, the ſame proportion holds with great accuracy, according to the dimenſions of their orbs, as determined by the obſervations of the beſt Aſtronomers.

P. 209, 211, 212.

That the ſuperior planets are revolved about the Sun, and by radii drawn to the Sun, deſcribe area's proportional to the times.

THAT Mars is revolved about the Sun, is demonſtrated from the phaſes which it ſhews, and the proportion of its apparent diameters. For from its appearing full near conjunction with the Sun, and gibbous in its quadratures, it is certain that it ſurrounds the Sun. [*]

And ſince its diameter appears about five times greater, when in oppoſition to the Sun, than when in conjunction therewith, and its diſtance from the Earth is reciprocally as its apparent diameter, that diſtance will be about five times leſs, when in oppoſition to, than when in conjunction with, the Sun. But in both caſes its diſtance from the Sun will be near about the ſame with the diſtance which is inferred from its gibbous appearance in the quadratures. And as it encompaſſes the Sun at almoſt equal diſtances, but in reſpect of the Earth is very unequally diſtant; ſo by radii drawn to the Sun, it deſcribes area's nearly uniform; but by radii drawn to the Earth, it is ſometimes ſwift, ſometimes ſtationary, and ſometimes retrograde.

That Jupiter, in a higher orb than Mars, is likewiſe revolved about the Sun, with a motion nearly equable, as well in diſtance as in the area's deſcribed, I infer thus:

Mr. *Flamſteed* aſſured me by letters, that all the eclipſes of the innermoſt ſatellite, which hitherto have been well obſerved, do agree with his theory ſo nearly, as never to differ therefrom by two minutes of time; that in the outmoſt, the errour is little greater; in the outmoſt but one, ſcarcely three times greater; that in the innermoſt but one, the difference is indeed much greater, yet ſo as to agree as nearly with his computations, as the Moon does with the common tables. And that he computes thoſe eclipſes only from the mean motions corrected by the equation of light diſcovered and introduced by Mr. *Romer*. Suppoſing then that the theory differs by a leſs errour than that of $2'$ from the motion of the outmoſt ſatellite as hitherto deſcribed; and taking as the periodic time $16^{d}\ 18^{h}\ 5'\ 13''$ to $2'$ in time, ſo is the whole circle or $360°$ to the arc $1'\ 48''$; the errour of Mr. *Flamſteed's* *ſteed's* computation reduced to the ſatellite's orbit, will be leſs than 1′ 48″; that is, the longitude of the ſatellite, as ſeen from the center of Jupiter, will be determined with a leſs errour than 1′ 48″. But when the ſatellite is in the middle of the ſhadow, that longitude is the ſame with the heliocentric longitude of Jupiter. And therefore the hypotheſis which Mr. *Flamſteed* follows, *viz.* the *Copernican*, as improved by *Kepler*, and (as to the motion of Jupiter) lately corrected by himſelf, rightly repreſents that longitude within a leſs errour than 1′ 48″. But by this longitude, together with the geocentric longitude, which is always eaſily found, the diſtance of Jupiter from the Sun is determined: Which muſt therefore be the very ſame with that which the hypotheſis exhibits. For that greateſt errour of 1′ 48″ that can happen in the heliocentric longitude is almoſt inſenſible, and quite to be neglected, and perhaps may ariſe from ſome yet undiſcovered eccentricity of the ſatellite. But ſince both longitude and diſtance are rightly determined, it follows of neceſſity, that Jupiter, by radii drawn to the Sun, deſcribes area's ſo conditioned as the hypotheſis requires, that is, proportional to the times.

And the ſame thing may be concluded of Saturn from his ſatellite by the obſervations of Mr. *Huygens* and Dr. *Halley*; though a longer ſeries of obſervations is yet wanting to confirm the thing, and to bring it under a ſufficiently exact computation.

P. 211.

That the force which governs the ſuperiour Planets is directed, not to the Earth, but to the Sun.

FOR if Jupiter was viewed from the Sun, it would never appear retrograde nor ſtationary, as it is ſeen ſometimes from the Earth, but always to go forward with a motion nearly uniform. And from the very great inequality of its apparent geocentric motion, we infer (by *Prop.* 3. *Cor.* 4.) that the force by which Jupiter is turned out of a rectilinear courſe, and made to revolve in an orb, is not directed to the center of the Earth. And the ſame argument holds good in Mars and in Saturn. Another center of theſe forces is therefore to be looked for (by *Prop.* 2 and 3, and the *Corollaries* of the latter) about which the area's deſcribed by radii intervening, may be equable. And that this is the Sun, we have proved already in Mars and Saturn nearly, but accurately enough in Jupiter. [*] It may be alledged that the Sun and Planets are impelled by ſome other force, equally and in the direction of parallel lines. But by ſuch a force (by *Cor.* 6. of the Laws of Motion) no change would happen in the ſituation of the Planets one to another, nor any ſenſible effect follow; but our buſineſs is with the cauſes of ſenſible effects. Let us therefore neglect every ſuch force as imaginary and precarious, and of no uſe in the phænomena of the heavens; and the whole remaining force by which Jupiter is impelled, will be directed (by *Prop.* 3. *Cor.* 1.) to the center of the Sun.

That the circumſolar force throughout all the regions of the Planets decreaſeth in the duplicate proportion of the diſtances from the Sun.

THE diſtances of the Planets from the Sun come out the ſame, whether, with *Tycho*, we place the Earth in the center of the ſyſtem, or the Sun with *Copernicus*: and we have already proved that theſe diſtances are true in Jupiter.

P. 210.

Kepler and *Bullialdus* have, with great care, determined the diſtances of the Planets from the Sun. And hence it is that their tables agree beſt with the heavens. And in all the Planets, in Jupiter and Mars, in Saturn and the Earth, as well as in Venus and Mercury, the cubes of their diſtances are as the ſquares of their periodic times; and therefore (by *Cor.* 6. *Prop.* 4.) the centripetal circum-ſolar force, throughout all the planetary regions, decreaſes in the duplicate proportion of the diſtances from the Sun. In examining this proportion, we are to uſe the mean diſtances, or the tranſverſe ſemi-axes of the orbits (by *Prop.* 15.) and to neglect thoſe little fractions, which, in defining the orbits, may have aroſe from the inſenſible errours of obſervation, or may be aſcribed to other cauſes, which we ſhall afterwards explain. And thus we ſhall always find the ſaid proportion to hold exactly. For the diſtances of Saturn, Jupiter, Mars, the Earth, Venus and Mercury from the Sun, drawn from the obſervations of Aſtronomers, are, according to the computation of *Kepler*, as the numbers 951000,519650,152350, 100000,72400,38806; by the computation of *Bullialdus*, as the numbers, 954198,522520,152350,100000,72398, 38585; and from the periodic times they come out 953806,520116,152399, 100000,72333,38710. Their diſtances, according to *Kepler* and *Bullialdus*, ſcarcely differ by any ſenſible quantity, and where they differ moſt the diſtances drawn from the periodic times fall in between them. [*]

THAT the circum-terreſtrial force likewiſe decreaſes in the duplicate proportion of the diſtances, thus I infer.

That the circum-terreſtrial force decreaſes in the duplicate proportion of the diſtances from the Earth proved in the hypotheſis of the Earth's being at reſt.

The mean diſtance of the Moon from the center of the Earth, is in ſemi-diameters of the Earth, according to *Ptolomy*, *Kepler* in his *Ephemerides*, *Bullialdus*, *Hevelius* and *Ricciolus* 59; according to *Flamſteed* $59\frac{1}{3}$, according to *Tycho* $56\frac{1}{2}$, to *Vendelin* 60, to *Copernicus* $60\frac{1}{3}$, to *Kircher* $62\frac{1}{2}$.

P. 215, 216, 217, 218.

But *Tycho* and all that follow his tables of refraction, making the refractions of the Sun and Moon (altogether againſt the nature of light) to exceed thoſe of the fixed Stars, and that by about four or five minutes in the horizon, did thereby augment the horizontal parallax of the Moon, by about the like number of minutes; that is, by about the 12th or 15th part of the whole parallax. Correct this errour, and the diſtance will become 60 or 61 ſemi-diameters of the Earth, nearly agreeing with what others have determined.

Let us then aſſume the mean diſtance of the Moon, 60 ſemi-diameters of the Earth, and its periodic time in reſpect of the fixed Stars, $27^{d}\ 7^{h}\ 43'$, as Aſtronomers have determined it. And (by *Cor.* 6. *Prop.* 4.) a body revolved in our air, near the ſurface of the Earth ſuppoſed at reſt, by means of a centripetal force, which ſhould be, to the ſame force at the diſtance of the Moon, in the reciprocal duplicate proportion of the diſtances from the center of the Earth, that is, as 3600 to 1, would (ſecluding the reſiſtance of the air) compleat a revolution in 1 h 24′ 27″.

Suppoſe the circumference of the Earth to be 123249600 *Paris* feet, as has been determined by the late menſuration of the *French*; then the ſame body, deprived of its circular motion, and falling by the impulſe of the ſame centripetal force as before, would, in one ſecond of time, deſcribe 15 $\frac{1}{12}$ *Paris* feet. *Vid. p.* 240.

This we infer by a calculus formed upon *Prop.* 36, and it agrees with what we obſerve in all bodies about the Earth. For by the experiments of pendulums, and a computation raiſed thereon, Mr. *Huygens* has demonſtrated that bodies falling by all that centripetal force, with which (of whatever nature it is) they are impelled near the ſurface of the Earth, do, in one ſecond of time, deſcribe 15 $\frac{1}{12}$ *Paris* feet.

The ſame proved in the hypotheſis of the Earth's motion.

BUT if the Earth is ſuppoſed to move, the Earth and Moon together (by *Cor.* 4. of the *Laws of Motion,* and *Prop.* 57.) will be revolved about their common center of gravity. And the Moon (by *Prop.* 60.) will in the ſame periodic time, $27^{d}\ 7^{h}\ 43'$, with the ſame circum-terreſtrial force, diminiſhed in the duplicate proportion of the diſtance, deſcribe an orbit, whoſe ſemi-diameter is to the ſemi-diameter of the former orbit, that is, to 60 ſemi-diameters of the Earth, as the ſum of both the bodies of the Earth and Moon to the firſt of two mean proportionals between this ſum and the body of the Earth; that is, if we ſuppoſe the Moon (on account of its mean apparent diameter $31\frac{1}{2}'$) to be about $\frac{1}{42}$ of the Earth, as 43 to $\sqrt[3]{42+\overline{43}|^{2}}$, or as about 128 to 127. And therefore the ſemi-diameter of the orbit, that is, the diſtance between the centers of the Moon and Earth, will in this caſe be $60\frac{1}{2}$ ſemi-diameters of the Earth, almoſt the ſame with that aſſigned by *Copernicus,* which the *Tychonic* obſervations by no means diſprove. And therefore the duplicate proportion of the decrement of the force holds good in this diſtance. I have neglected the increment of the orbit, which ariſes from the action of the Sun, as inconſideconſiderable. But if that is ſubducted the true diſtance will remain, about $60\frac{4}{9}$ [✳] ſemidiameters of the Earth.

P. 214.

The decrement of the forces in the duplicate proportion of the diſtances from the Earth and Planets, proved from the eccentricity of the Planets, and the very ſlow motion of their apſes.

BUT further, this proportion of the decrement of the forces is confirmed from the eccentricity of the Planets, and the very ſlow motion of their apſes. For (by the *Corollaries* of *Prop.* 45.) in no other proportion, could the circum-ſolar planets, once in every revolution deſcend to their leaſt, and once aſcend to their greateſt diſtance from the Sun, and the places of thoſe diſtances remain immovable. A ſmall errour from the duplicate proportion, would produce a motion of the apſes, conſiderable in every [✳] revolution, but in many enormous.

But now after innumerable revolutions, hardly any ſuch motion has been perceived in the orbs of the circum-ſolar planets. Some Aſtronomers affirm, that there is no ſuch motion, others reckon it no greater than what may eaſily ariſe from the cauſes hereafter to be aſſigned, and is of no moment in the preſent queſtion.

P. 214, 215. We may even neglect the motion of the Moon's apſe, which is far greater than in the circum-ſolar planets, amounting in every revolution to three degrees. And from this motion it is demonſtrable, that the circum-terreſtrial force decreaſes in no leſs than the duplicate, but far leſs than the triplicate proportion of the diſtance. For if the duplicate proportion was gradually changed into the triplicate, the motion of the apſe would thereby increaſe to infinity; and therefore, by a very ſmall mutation, would exceed the motion of the Moon's apſe. This ſlow motion ariſes from the action of the circum-ſolar force, as we ſhall afterwards explain. But ſecluding this cauſe, the apſe or apogeon of the Moon will be fixed, and the duplicate proportion of the decreaſe of the circum-terreſtrial force in different diſtances from the Earth, will accurately take place.

P. 227.

The quantity of the forces tending towards the ſeveral planets. The circum-ſolar very great.

NOW that this proportion has been eſtabliſhed, we may compare the forces of the ſeveral planets among themſelves.

In the mean diſtance of Jupiter from the Earth, the greateſt elongation of the outmoſt ſatellite from Jupiter's center (by the obſervations of Mr. *Flamſteed*) is 8′ 13″. And therefore the diſtance of the ſatellite from the center of Jupiter, is to the mean diſtance of Jupiter from the center of the Sun, as 124 to 52012, but to the mean diſtance of Venus from the center of the Sun, as 124 to 7234. And their periodic times are $16\frac{3}{4}^{d}$ and $224\frac{3}{2}^{d}$. And from hence (according to *Cor.* 2. *Prop.* 4.) dividing the diſtances by the ſquares of the times, we infer that the force by which the ſatellite is impelled towards Jupiter, is to the force by which Venus is impelled towards the Sun, as 442 to 143. And if we diminiſh the force, by which the ſatellite is impelled, in the duplicate proportion of the diſtance 124 to 7234, we ſhall have the circum-jovial force, in the diſtance of Venus from the Sun, to the circum-ſolar force by which Venus is impelled, as $\frac{13}{100}$ to 143, or as 1 to 1100. Wherefore at equal diſtances, the circum-ſolar force is 1100 times greater than the circum-jovial.

And by the like computation, from the periodic time of the ſatellite of Saturn 15^{d} 22^{h} and its greateſt elongation from Saturn, while that planet is in its mean diſtance from us, 3′ 20″. it follows that the diſtance of this ſatellite from Saturn's center, is to the diſtance of Venus from the Sun, as $92\frac{2}{5}$ to 7234; and from thence that the abſolute circum-ſolar force is 2360 times greater than the abſolute circum-ſaturnial.

The circum-terreſtrial force very ſmall.

FROM the regularity of the heliocentric and irregularity of the geocentric motions of Venus, of Jupiter, and the other planets, it is evident (by *Cor.* 4. *Prop.* 3.) that the circum-terreſtrial force, compared with the circum-ſolar, is very ſmall.

Ricciolus and *Vendelin* have ſeverally tried to determine the Sun's parallax, from the Moon's dichotomies obſerved by the teleſcope, and they agree that it does not exceed half a minute.

Kepler, from *Tycho*'s obſervations and his own, found the parallax of Mars inſenſible, even in oppoſition to the Sun, when that parallax is ſomething greater than the Sun's.

Flamſteed attempted the ſame parallax with the micrometer in the perigeon poſition of Mars, but never found it above 25″; and thence concluded the Sun's parallax at moſt 10″.

Whence it follows, that the diſtance of the Moon from the Earth bears no greater proportion to the diſtance of the Earth from the Sun, than 29 to 10000; nor to the diſtance of Venus from the Sun, than 29 to 7233.

From which diſtances, together with the periodic times, by the method above explained it is eaſy to infer, That the abſolute circum-ſolar force is greater than the abſolute circum-terreſtrial force at leaſt 229400 times.

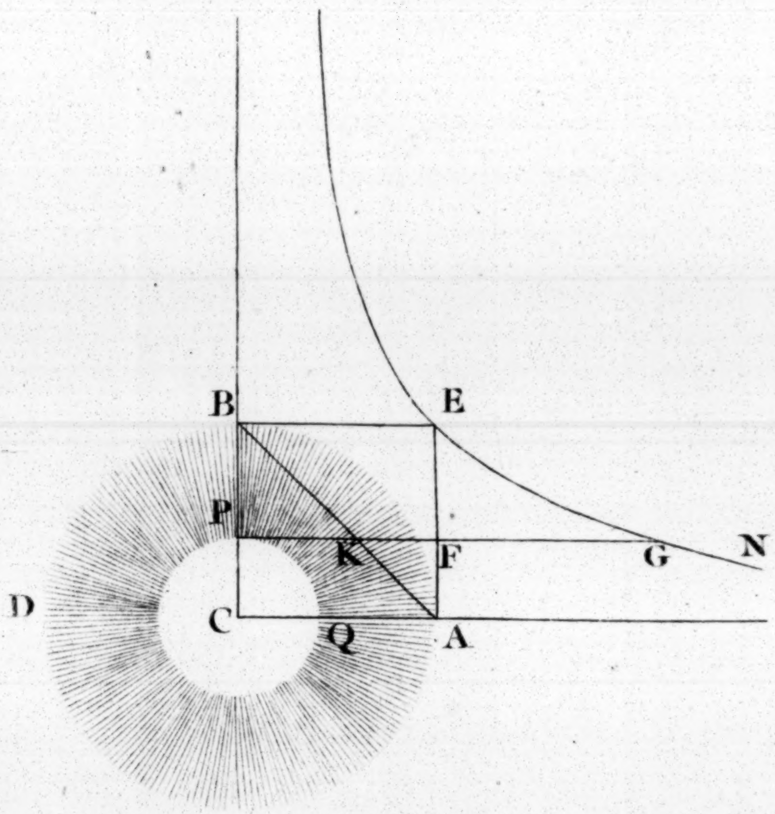
And though we were only certain, from the obſervations of *Ricciolus* and *Vendelin*, that the Sun's parallax was leſs than half a minute, yet from this it will follow, that the abſolute circum-ſolar force exceeds the abſolute circum-terreſtrial force 8500 times. [*]

The apparent diameters of the planets.

By the like Computations I happened to diſcover an analogy that is obſerved between the forces and the bodies of the Planets. But before I explain this analogy, the apparent diameters of the Planets in their mean diſtances from the Earth, muſt be firſt determined.

P. 207. Mr. *Flamſteed*, by the micrometer, meaſured the diameter of Jupiter 40″ or 41″, the diameter of Saturn's ring 50″, and the diameter of the Sun about 32′ 13″. [*]

P. 209. But the diameter of Saturn is to the diameter of the ring, according to Mr. *Huygens* and Dr. *Halley*, as 4 to 9; according to *Galletius*, as 4 to 10; and according to *Hook* (by a teleſcope of 60 feet) as 5 to 12. And from the mean pro-



Page 29.

proportion 5 to 12, the diameter of Saturn's body is inferred about 21″. [*]

The correction of the apparent diameters.

SUCH as we have ſaid are the apparent magnitudes. But becauſe of the unequal refrangibility of light, all lucid points are dilated by the teleſcope, and in the focus of the object-glaſs poſſeſs a circular ſpace, whoſe breadth is about the 50th part of the aperture of the glaſs.

It is true, that towards the circumference, the light is ſo rare as hardly to move the ſenſe; but towards the middle, where it is of greater denſity, and is ſenſible enough, it makes a ſmall lucid circle, whoſe breadth varies according to the ſplendour of the lucid point, but is generally about the 3d, or 4th, or 5th part of the breadth of the whole.

Let ABD repreſent the circle of the whole light, PQ the ſmall circle of the denſer and clearer light, C the center of both; CA, CB ſemi-diameters of the greater circle containing a right angle at C; ACBE the ſquare comprehended under theſe ſemi-diameters, AB the diagonal of that ſquare; EGH an hyperbola with the center C and aſymptotes CA, CB; PG a perpendicular erected from any point P of the line BC, and meeting the hyperbola in G, and the right lines AB, AE in K and F: and the denſity of the light in any place P, will, by my computation, be as the line FG, and therefore at the center infinite, but near the circumference very ſmall. And the whole light within the ſmall circle PQ is to the whole without, as the area of the quadrilateral figure CAKP to the triangle PKB. And we are to underſtand the ſmall circle PQ to be there terminated, where FG, the denſity of the light, begins to be leſs than what is required to move the ſenſe.

Hence it was that at the diſtance of 191382 feet, a fire of 3 feet in diameter, through a teleſcope of 3 feet, appeared to Mr. *Picart* of 8″ in breadth, when it ſhould have appeared only of 3″ 14‴. And hence it is that the brighter fixed Stars appear through the teleſcope, as of 5″ or 6″ in diameter, and that with a good full light; but with a fainter light they appear to run out to a greater breadth. Hence likewiſe it was that *Hevelius*, by diminiſhing the aperture of the teleſcope, did cut off a great part of the light towards the circumference, and brought the diſc of the Star to be more diſtinctly defined, which though hereby diminiſhed, did yet appear as of 5″ or 6″ in diameter. But Mr. *Huygens*, only by clouding the eye-glaſs with a little ſmoak, did ſo effectually extinguiſh this ſcattered light, that the fixed Stars appeared as meer points, void of all ſenſible breadth. Hence alſo it was that Mr. *Huygens*, from the breadth of bodies interpoſed to intercept the whole light of the Planets, reckoned their diameters greater than others have meaſured them by the micrometer. For the ſcattered light, which could not be ſeen before for the ſtronger light of the Planet, when the Planet is hid, appears every way further ſpread. Laſtly, from hence it is that the Planets appear ſo ſmall in the diſc of the Sun, being leſſened by the dilated light. For to *Hevelius*, *Galletius*, and Dr. *Halley*, Mercury did not ſeem to exceed 12″ or 15″. And Venus appeared to Mr. *Crabtrie* only 1′ 3″, to *Horrox* but 1′ 12″, though by the menſurations of *Hevelius* and *Hugenius*, without the Sun's diſc, it ought to have been ſeen at leaſt 1′ 24″. Thus the apparent diameter of the Moon, which in 1684, a few days both before and after the Sun's eclipſe, was meaſured at the obſervatory of *Paris* 31′ 30″; in the eclipſe it ſelf did not ſeem to exceed 30′ or 30′ 05″. And therefore the diameters of the Planets are to be diminiſhed, when without the Sun, and to be augmented when within it by ſome ſeconds. But the errours ſeem to be leſs than uſual in the menſurations that are made by the micrometer. So from the diameter of the ſhadow, determined by the eclipſes of the ſatellites, Mr. *Flamſteed* found that the ſemi-diameter of Jupiter, was to the greateſt elongation of the outmoſt ſatellite as 1 to 24,903. Wherefore ſince that elongation is 8′ 13″, the diameter of Jupiter will be 39½″; and rejecting the ſcattered light, the diameter, found by the micrometer 40″ or 41″, will be reduced to 39½″. And the diameter of Saturn 21″, is to be diminiſhed by the like correction, and to be reckoned 20″ or ſomething leſs. But (if I am not miſtaken) the diameter of the Sun, becauſe of its ſtronger light, is to be diminiſhed ſomething more, and to be reckoned about 32′ or 32′ 6″.

P. 229.

Why the denſity is greater in ſome of the Planets, and leſs in others; but the forces in all are as their quantities of matter.

THAT bodies ſo different in magnitude ſhould come ſo near to an analogy with their forces, is not without ſome myſtery.

It may be that the remoter Planets, for want of heat, have not thoſe metallic ſubſtances and ponderous minerals with which our Earth abounds; and that the bodies of Venus and Mercury, as they are more expoſed to the Sun's heat, are alſo harder baked and more compact.

For from the experiment of the burning-glaſs, we ſee that the heat increaſes with the denſity of light. And this denſity increaſes in the reciprocal duplicate proportion of the diſtance from the Sun.. From whence the Sun's heat in Mercury is proved to be ſeven-fold its heat in our ſummer ſeaſons. But with this heat our water boils; and thoſe heavy fluids, quickſilver and the ſpirit of vitriol, gently evaporate, as I have tried by the thermometer. And therefore there can be no fluids in Mercury, but what are heavy, and able to bear a great heat, and from which ſubſtances of great denſity may be nouriſhed.

And why not? if God has placed different bodies at different diſtances from the Sun, ſo as the denſer bodies always poſſeſs the nearer places, and each body enjoys a degree of heat ſuitable to its condition, and proper for its nouriſhment. From this conſideration it will beſt appear that the weights of all the Planets are one to another as their forces. [*]

But I ſhould be glad the diameters of the Planets were more accurately meaſured. And that may be done, if a lamp, ſet at a great diſtance, is made to ſhine through a circular hole, and both the hole and the light of the lamp are ſo diminiſhed that the ſpectrum may appear through the teleſcope juſt like the Planet, and may be defined by the ſame meaſure: Then the diameter of the hole will be to its diſtance from the objective glaſs, as the true diameter of the Planet to its diſtance from us. The light of the lamp may be diminiſhed by the interpoſition either of pieces of cloth, or of ſmoaked glaſs.

P. 221, 222, 223.

Another analogy between the forces and bodies, proved in the celeſtial bodies.

OF kin to the analogy we have been deſcribing, there is another obſerved between the forces and the bodies attracted. Since the action of the centripetal force upon the Planets decreaſes in the duplicate proportion of the diſtance, and the periodic time increaſes in the ſeſquiplicate thereof, it is evident that the actions of the centripetal force, and therefore the periodic times, would be equal in equal Planets, at equal diſtances from the Sun; and in equal diſtances of unequal Planets, the total actions of the centripetal force would be as the bodies of the Planets. For if the actions were not proportional to the bodies to be moved, they could not equally retract theſe bodies from the tangents of their orbs, in equal times: Nor could the motions of the ſatellites of Jupiter be ſo regular, if it was not that the circumſolar force was equally exerted upon Jupiter and all its ſatellites in proportion of their ſeveral weights. And the ſame thing is to be ſaid of Saturn in reſpect of its ſatellite, and of our Earth in reſpect of the Moon, as appears from *Cor.* 2 and 3. *Prop.* 65. And therefore at equal diſtances, the actions of the centripetal force is equal upon all the Planets, in proportion of their bodies, or of the quantities of matter in their ſeveral bodies: and for the ſame reaſon muſt be the ſame upon all the particles of the ſame ſize, of which the planet is compoſed. For if the action was greater, upon ſome ſort of particles than upon others, than in proportion to their quantity of matter, it would be alſo greater or leſs, upon the whole Planets, not in proportion of the quantity only, but likewiſe of the ſort of the matter more copiouſly found in one, and more ſparingly in another. [*]

P. 220, 221.

Proved in terreſtrial bodies.

In ſuch bodies as are found on our Earth of very different ſorts, I examined this analogy with great accuracy.

If the action of the circumterreſtrial force is proportional to the bodies to be

moved,

moved, it will (by the ſecond *Law of Motion*) move them with equal velocity in equal times, and will make all bodies let fall to deſcend through equal ſpaces in equal times; and all bodies hung by equal threads, to vibrate in equal times. If the action of the force was greater, the times would be leſs: If that was leſs, theſe would be greater.

But it has been long ago obſerved by others, that (allowance being made for the ſmall reſiſtance of the Air) all bodies deſcend through equal ſpaces in equal times. And, by the help of pendulums, that equality of times may be diſtinguiſhed to great exactneſs.

I tried the thing in gold, ſilver, lead, glaſs, ſand, common ſalt, wood, water and wheat. I provided two equal wooden boxes. I filled the one with wood, and ſuſpended an equal weight of gold (as exactly as I could) in the center of oſcillation of the other. The boxes, hung by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally expoſed to the reſiſtance of the air: And placing the one by the other, I obſerved them to play together, forwards and backwards, for a long while, with equal vibrations. And therefore (by *Cor.* 1 and 6. *Prop.* 24. *Book* II) the quantity of matter in the gold was to the quantity of matter in the wood, as the action of the motive force upon all the gold, to the action of the ſame upon all the wood, that is, as the weight of the one to the weight of the other.

And by theſe experiments, in bodies of the ſame weight, I could have diſcovered a difference of matter, leſs than the thouſandth part of the whole. [*]

The affinity of thoſe analogies.

SINCE the action of the centripetal force upon the bodies attracted, is, at equal diſtances, proportional to the quantities of matter in thoſe bodies, reaſon requires that it ſhould be alſo proportional to the quantity of matter in the body attracting.

P. 20, 37. Vol. I.

For all action is mutual, and (by the third *Law of Motion*) makes the bodies mutually to approach one to the other, and therefore muſt be the ſame in both bodies. It is true that we may conſider one body as attracting, another as attracted: But this diſtinction is more mathematical than natural. The attraction is really common of either to other, and therefore of the ſame kind in both. [*]

And coincidence.

AND hence it is that the attractive force is found in both. The Sun attracts Jupiter and the other Planets. Jupiter attracts its ſatellites.

And for the ſame reaſon, the ſatellites act as well one upon another as upon Jupiter, and all the Planets mutually one upon another.

And though the mutual actions of two Planets may be diſtinguiſhed and conſidered as two, by which each attracts the other; yet as thoſe actions are intermediate, they do not make two, but one operation between two terms. Two bodies may be mutually attracted, each to the other, by the contraction of a cord interpoſed. There is a double cauſe of action, to wit, the diſpoſition of both bodies, as well as a double action in ſo far as the action is conſidered as upon two bodies: But as betwixt two bodies it is but one ſingle one. It is not one action by which the Sun attracts Jupiter, and another by which Jupiter attracts the Sun: But it is one action by which the Sun and Jupiter mutually endeavour to approach each the other. By the action with which the Sun attracts Jupiter, Jupiter and the Sun endeavour to come nearer together (by the *third Law of Motion*) and by the action, with which Jupiter attracts the Sun, likewiſe Jupiter and the Sun endeavour to come nearer together: But the Sun is not attracted towards Jupiter by a two-fold action, nor Jupiter by a two-fold action towards the Sun: but it is one ſingle intermediate action, by which both approach nearer together.

Thus iron draws the load-ſtone, as well as the load-ſtone draws the iron: For all iron in the neighbourhood of the load-ſtone draws other iron. But the action betwixt the load-ſtone and iron is ſingle, and is conſidered as ſingle by the philoſophers. The action of iron upon the load-ſtone is indeed the action of the load-ſtone betwixt itſelf and the iron, by which both endeavour to come nearer together; and ſo it manifeſtly appears: For if you remove the load-ſtone, the whole force of the iron almoſt ceaſes.

P. 37. Vol. I.

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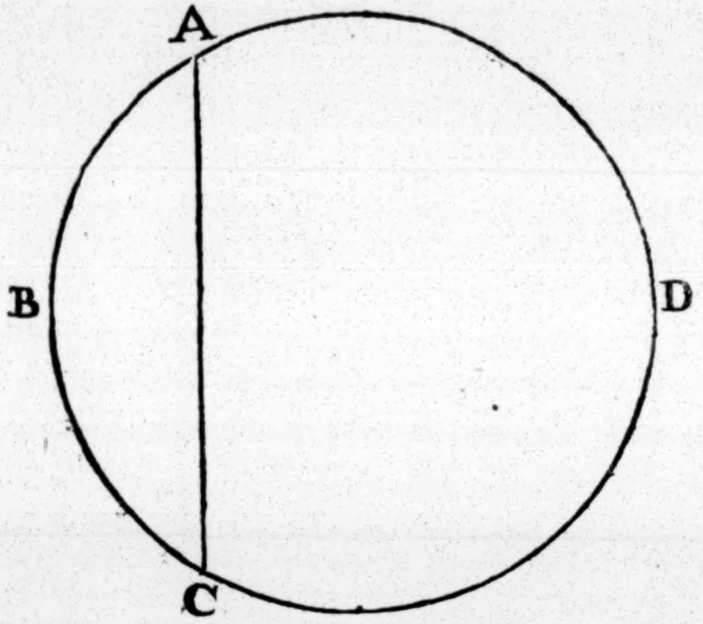
In this ſenſe it is that we are to conceive one ſingle action to be exerted betwixt two Planets, ariſing from the conſpiring natures of both. And this action ſtanding in the ſame relation to both, if it is proportional to the quantity of mattter in the one, it will be alſo proportional to the quantity of matter in the other.

PERHAPS it may be objected, that according to this philoſophy all bodies ſhould mutually attract one another, contrary to the evidence of experiments in terreſtrial

P. 226.

That the forces of ſmall bodies are inſenſible.

ſtrial bodies. But I anſwer, that the experiments in terreſtrial bodies come to no account. For the attraction of homogeneous ſpheres near their ſurfaces, are (by *Prop.* 72) as their diameters. Whence a ſphere of one foot in diameter, and of a like nature to the Earth, would attract a ſmall body placed near its ſurface with a force 20000000 times leſs, than the Earth would do if placed near its ſurface. But ſo ſmall a force could produce no ſenſible effect. If two ſuch ſpheres were diſtant but by $\frac{1}{4}$ of an inch, they would not even in ſpaces void of reſiſtance, come together by the force of their mutual attraction in leſs than a months time. And leſs ſpheres will come together at a rate yet ſlower, viz. in the proportion of their diameters. Nay, whole mountains will not be ſufficient to produce any ſenſible effect. A mountain of an hemiſpherical figure, three miles high, and ſix broad, will not, by its attraction, draw the pendulum 2 minutes out of the true perpendicular: And it is only in the great bodies of the Planets that theſe forces are to be perceived, unleſs we may reaſon about ſmaller bodies in manner following. [*]



P. 37. Vol. I.

Which notwithſtanding, there are forces tending towards all terreſtrial bodies, proportional to their quantities of matter,

LET ABCD repreſent the globe of the Earth, cut, by any plane AC, into two parts ACB, and ACD. The part ACB bearing upon the part ACD preſſes it with its whole weight: Nor can the part ACD ſuſtain this preſſure and continue unmoved, if it is not oppoſed by an equal contrary preſſure. And therefore the parts equally preſs each other by their weights, that is, equally attract each other, according to the *third Law of Motion*; and if ſeparated and let go, would fall towards each other with velocities reciprocally as the bodies. All which we may try and ſee in the load-ſtone, whoſe attracted ed part does not propel the part attracting, but is only ſtopped and ſuſtained thereby.

Suppoſe now that ACB repreſents ſome ſmall body on the Earth's ſurface; then becauſe the mutual attractions of this particle and of the remaining part ACD of the Earth towards each other, are equal; but the attraction of the particle towards the Earth (or its weight) is as the matter of the particle (as we have proved by the experiment of the pendulums) the attraction of the Earth towards the particle, will likewiſe be as the matter of the particle. And therefore the attractive forces of all terreſtrial bodies will be as their ſeveral quantities of matter.

P. 223.

Proved that the ſame forces tend towards the celeſtial bodies.

THE forces, which are as the matter in terreſtrial bodies of all forms, and therefore are not mutable with the forms, muſt be found in all ſorts of bodies whatſoever, celeſtial as well as terreſtrial, and be in all proportional to their quantities of matter, becauſe among all there is no difference of ſubſtance, but of modes and forms only. [*] But in the celeſtial bodies, the ſame thing is likewiſe proved thus. We have ſhewed, that the action of the circumſolar force upon all the Planets (reduced to equal diſtances) is as the matter of the Planets; That the action of the circumjovial force upon the ſatellites of Jupiter obſerves the ſame law; and the ſame thing is to be ſaid of the attraction of all the Planets towards every Planet: But thence it follows (by *Prop.* 69) that their attractive forces are as their ſeveral quantities of matter.

That from the ſurfaces of the Planets, reckoning outward, their forces decreaſe in the duplicate; but, reckoning inward, in the ſimple proportion of the diſtances from their centers.

As the parts of the Earth mutually attract one another, ſo do thoſe of all the Planets. If Jupiter and its ſatellites were brought together, and formed into one globe, without doubt they would continue mutually to attract one another as before. And on the other hand, if the body of Jupiter was broke into more globes, to be ſure, theſe would no leſs attract one another than they do the ſatellites now. From theſe attractions it is that the bodies of the Earth, and all the Planets, affect a ſpherical figure, and that their parts cohere, and are not diſperſed through the Æther. But we have before proved that theſe forces ariſe from the univerſal nature of matter, and that therefore the force of any whole globe is made up of the ſeveral forces of all its parts. And from thence it follows (by *Cor.* 3. *Prop.* 74) that the force of every par- P. 226.

particle decreaſes in the duplicate proportion of the diſtance from that particle; and (by *Prop.* 73 and 75) that the force of an entire globe, reckoning from the ſurface outwards, decreaſes in the duplicate, but reckoning inwards, in the ſimple proportion of the diſtances, from the centers, if the matter of the globe be uniform. And though the matter of the globe, reckoning from the center towards the ſurface, is not uniform, yet the decreaſe in the duplicate proportion of the diſtance outwards would (by *Prop.* 76) take place, provided that difformity is ſimilar in places round about at equal diſtances from the center. And two ſuch globes will (by the ſame *Propoſition*) attract one the other with a force decreaſing in the duplicate proportion of the diſtance between their centers. [*]

P. 219, 229.

P. 226, 227.

The quantities of the forces and of the motions ariſing in the ſeveral caſes.

WHEREFORE the abſolute force of every globe is as the quantity of matter which the globe contains: But the motive force by which every globe is attracted towards another, and which, in terreſtrial bodies, we commonly call their weight, is as the content under the quantities of matter in both globes applied to the ſquare of the diſtance between their centers (by *Cor*. 4. *Prop*. 76.) to which force the quantity of motion, by which each globe in a given time will be carried towards the other, is proportional. And the accelerative force, by which every globe according to its quantity of matter is attracted towards another, is as the quantity of matter in that other globe applied to the ſquare of the diſtance between the centers of the two (by *Cor*. 2. *Prop*. 76.) to which force, the velocity, by which the attracted globe will, in a given time, be carried towards the other, is proportional. And from theſe principles well underſtood, it will be now eaſy to determine the motions of the celeſtial bodies among themſelves.

That all the Planets revolve about the Sun.

FROM comparing the forces of the Planets one with another, we have above ſeen that the circumſolar does more than a thouſand times exceed all the reſt. But by the action of a force ſo great, it is unavoidable but that all bodies within, nay and far beyond, the bounds of the planetary ſyſtem, muſt deſcend directly to the Sun, unleſs by other motions they are impelled towards other parts: Nor is our Earth to be excluded from the number of ſuch bodies. For certainly the Moon is a body of the ſame nature with the

Planets, and ſubject to the ſame attractions with the other Planets, ſeeing it is by the circumterreſtrial force that it is retained in its orbit. But that the Earth and Moon are equally attracted towards the Sun, we have above proved: We have likewiſe before proved, that all bodies are ſubject to the ſaid common laws of attraction. Nay, ſuppoſing any of thoſe bodies to be deprived of its circular motion about the Sun, by having its diſtance from the Sun we may find (by *Prop.* 36) in what ſpace of time it would, in its deſcent, arrive at the Sun; to wit, in half that periodic time in which the body might be revolved at one half of its former diſtance; or, in a ſpace of time that is to the periodic time of the planet as 1 to $4\sqrt{2}$. As that Venus in its deſcent would arrive at the Sun in the ſpace of 40 days, Jupiter in the ſpace of two years and one month; and the Earth and Moon together in the ſpace of 66 days and 19 hours. But ſince no ſuch thing happens, it muſt needs be that thoſe bodies are moved towards other parts. Nor is every motion ſufficient for this purpoſe. To hinder ſuch a deſcent, a due proportion of velocity is required. And hence depends the force of the argument drawn from the retardation of the motions of the Planets.

P. 5. *Vol.* I.

Unleſs the circumſolar force decreaſed in the duplicate ratio of their increaſing ſlowneſs, the exceſs thereof would force thoſe bodies to deſcend to the Sun. For inſtance, if the motion (*cæteris paribus*) was retarded by one half, the planet would be retained in its orb by one fourth of the former circumſolar force, and by the exceſs of the other three fourths would deſcend to the Sun. And [*] therefore the Planets (Saturn, Jupiter, Mars, Venus, and Mercury) are not really retarded in their perigees, nor become really ſtationary, or regreſſive with ſlow motions. All theſe are but apparent, and the abſolute motions, by which the Planets continue to revolve in their orbits, are always direct and nearly equable. But that ſuch motions are performed about the Sun, we have already proved; and therefore the Sun, as the center of the abſolute motions, is quieſcent. For we can by no means allow quieſcence to the Earth, leſt the Planets in their perigees ſhould indeed be truly retarded, and become truly ſtationary and regreſſive, and ſo for want of motion ſhould deſcend to the Sun. But further, ſince the Planets (Venus, Mars, Jupiter and the reſt) by radii drawn to the Sun deſcribe regular orbits, and area's (as we have ſhewed) nearly and to ſenſe proportional to the times, it follows (by *Prop.* 3, and *Cor.* 3, *Prop.* 65) that the Sun is moved with no notable force, unleſs perhaps with ſuch as all the Planets are equally moved with, according to their ſeveral quantities of matter, in parallel lines, and ſo the whole ſyſtem is transferred in right lines. Reject that tranſlation of the whole ſyſtem, and the Sun will be almoſt quieſcent in the center thereof. If the Sun was revolved about the Earth, and carried the other Planets round about itſelf, the Earth ought to attract the Sun with a great force, but the circumſolar Planets with no force producing any ſenſible effect, which is contrary to *Prop.* 3, *Cor.* 65. Add to this, that if hitherto the Earth, becauſe of the gravitation of its parts, has been placed by moſt authors in the lowermoſt region of the Univerſe; now for better reaſon, the Sun poſſeſſed of a centripetal force exceeding our terreſtrial gravitation a thouſand times and more, ought to be depreſſed into the lowermoſt place, and to be held for the center of the ſyſtem. And thus the true diſpoſition of the whole ſyſtem will be more fully and more exactly underſtood.

BECAUSE

P. 232, 233.

That the common center of gravity of all the Planets is quieſcent. That the Sun is agitated with a very ſlow motion. This motion defined.

BECAUSE the fixed Stars are quieſcent one in reſpect of another, we may conſider the Sun, Earth, and Planets as one ſyſtem of bodies carried hither and thither by various motions among themſelves; and the common center of gravity of all (by *Cor.* 4. of the *Laws of motion*) will either be quieſcent, or move uniformly forward in a right line: In which caſe the whole ſyſtem will likewiſe move uniformly forward in right lines. But this is an hypotheſis hardly to be admitted. And therefore ſetting it aſide, that common center will be quieſcent: And from it the Sun is never far removed. The common center of gravity of the Sun and Jupiter falls on the ſurface of the Sun. And though all the Planets were placed towards the ſame parts from the Sun with Jupiter, the common center of the Sun and all of them would ſcarcely recede twice as far from the Sun's center. And therefore though the Sun, according to the various ſituation of the Planets, is variouſly agitated and always wandering to and fro with a ſlow motion of libration, yet it never recedes one entire diameter of its own body from the quieſcent center of the whole ſyſtem. But from the weights of the Sun [*]

and Planets above determined, and the ſituation of all among themſelves, their common center of gravity may be found, and this being given, the Sun's place to any ſuppoſed time may be obtained.

P. 234.

That the Planets nevertheleſs are revolved in ellipſes, having their foci in the Sun; and by radii drawn to the Sun deſcribe area's proportional to the times.

ABOUT the Sun thus librated the other Planets are revolved in elliptic orbits, and, by radii drawn to the Sun, deſcribe area's nearly proportional to the times, as is explained in *Prop.* 65. If the Sun was quieſcent, and the other Planets did not act mutually one upon another, their orbits would be elliptic, and the area's exactly proportional to the times. (by *Prop.* 11, and *Cor.* 1. *Prop.* 13.) But the actions of the Planets among themſelves, compared with the actions of the Sun on the Planets, are of no moment, and produce no ſenſible errours. And thoſe errours are leſs in revolutions about the Sun agitated in the manner but now deſcribed, than if thoſe revolutions were made about the Sun quieſcent (by *Prop.* 66, and *Cor.* *Prop.* 68) eſpecially if the focus of every orbit is placed in the common center of gravity of all the lower included Planets; *viz.* the focus of the orbit of Mercury, in the center of the Sun; the focus of the orbit of Venus, in the common center of gravity of Mercury and the Sun; the focus of the orbit of the Earth, in the common center of gravity of Venus, Mercury and the Sun; and ſo of the reſt. And by this means the foci of the orbits of all the Planets except Saturn, will not be ſenſibly removed from the center of the Sun, nor will the focus of the orbit of Saturn recede ſenſibly from the common center of gravity of Jupiter and the Sun. And therefore aſtronomers are not far from the truth, when they reckon the Sun's center the common focus of all the planetary orbits. In Saturn it ſelf, the errour thence ariſing does not exceed 1′. 45″. And if its orbit, by placing the focus thereof in the common center of gravity of Jupiter and the Sun ſhall happen to agree better with the phænomena, from thence all that we have ſaid will be farther confirmed. [*]

Of the dimenſions of the orbits, and of the motions of their aphelions and nodes.

IF the Sun was quieſcent and the Planets did not act one on another, the aphelions and nodes of their orbits would likewiſe (by *Prop.* 1. 11, and *Cor. Prop.* 13) be quieſcent. And the longer axes of their elliptic orbits would (by *Prop.* 15) be as the cubic roots of the ſquares of their periodic times: And therefore from the given periodic times would be alſo given. But thoſe times are to be meaſured not from the equinoctial points, which are moveable, but from the firſt Star of Aries. Put the ſemi-axe of the Earth's orbit 100000, and the ſemi-axes of the orbits of Saturn, Jupiter, Mars, Venus and Mercury, from their periodic times will come out 953806, 520116, 152399, 72333, 38710 reſpectively. But from the Sun's motion every ſemi-axe is encreaſed (by *Prop.* 60) by about one third of the diſtance of the Sun's center from the common center of gravity of P. 236, 237. the Sun and Planet. And from the actions of the exterior Planets on the interiour, the periodic times of the interiour are ſomething protracted, though ſcarcely by any ſenſible quantity; and their aphelions are transferred (by *Cor.* 6 and 7. *Prop.* 66) by very ſlow motions [*] in *conſequentia*. And on the like account the periodic times of all, eſpecially of the exteriour Planets, will be prolonged by the actions of the comets, if any ſuch there are, without the orb of Saturn, and the aphelions of all will be thereby carried forwards in *conſequentia*. But from the progreſs of the aphelions, the regreſs of the nodes follows (by *Cor.* 11, 13. *Prop.* 66.) And if the plain of the Ecliptic is quieſcent, the regreſs of the nodes (by *Cor.* 16. *Prop.* 66) will be to the progreſs of the aphelion in every orbit, as the regreſs of the nodes of the Moon's orbit to the progreſs of its apogæon nearly, that is, as about 10 to 21. But aſtronomical obſervations ſeem to confirm a very ſlow progreſs of the aphelions, and a regreſs of the nodes in reſpect of the fixed Stars. And hence it is probable that there are comets in the regions beyond the Planets, which revolving in very eccentric orbs, quickly fly through their perihelion parts, and by an exceeding ſlow motion in their aphelions, ſpend almoſt their whole time in the regions beyond the Planets; as we ſhall afterwards explain more at large.

P. 252, 253, 254.

All the motions of the Moon that have hitherto been obſerved by aſtronomers, derived from the foregoing principles.

THE Planets thus revolved about the Sun, may at the ſame time carry others revolving about themſelves as Satellites or Moons, as appears by *Prop.* 66. But from the action of the Sun, our Moon muſt move with greater velocity, and, by a radius drawn to the Earth, deſcribe an area greater for the time; it muſt have its orbit leſs curve, and therefore approach nearer to the Earth, in the ſyzygies than in the quadratures, except in ſo far as the motion of eccentricity hinders thoſe effects. For the eccentricity is greateſt when the Moon's apogæon is in the ſyzygies, and leaſt when the ſame is in the quadratures; and hence it is that the perigeon-Moon is ſwifter and nearer to us, but the apogæon-Moon ſlower and farther from us, in the ſyzygies than in the quadratures. But further, the apogæon has a progreſſive, and the nodes a regreſſive motion, both unequable. For the apogæon is more ſwiftly progreſſive in its ſyzygies, more ſlowly regreſſive in its quadratures, and by the exceſs of its progreſs above its regreſs is yearly transferred in *conſequentia*: but the nodes are quieſcent in their ſyzygies, and moſt ſwiftly regreſſive in their quadratures. But further ſtill, the greateſt latitude of the Moon is greater in its quadratures than in its ſyzygies; and the mean motion ſwifter in the aphelion of the Earth than in its perihelion. More inequalities in the Moon's motion have not hitherto been taken notice of by aſtronomers: But [*] all theſe follow from our principles in *Cor.* 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13. *Prop.* 66. and are known really to exiſt in the heavens. And this may be ſeen in that moſt ingenious, and, if I miſtake not, of all the moſt accurate, hypotheſis

ſis of Mr. *Horrox*, which Mr. *Flamſteed* has fitted to the heavens. But the aſtronomical hypotheſes are to be correcӡed in the motion of the nodes. For the nodes admit the greateſt equation or proſthaphæreſis in their octants, and this inequality is moſt conſpicuous, when the Moon is in the nodes, and therefore alſo in the octants, and hence it was that *Tycho*, and others after him, referred this inequality to the octants of the Moon, and made it menſtrual. But the reaſons by us adduced prove that it ought to be referred to the octants of the nodes, and to be made annual.

P. 253, 299 *to* 302.

As alſo ſome other unequable motions that hitherto have not been obſerved.

BESIDE thoſe inequalities taken notice of by aſtronomers, there are yet ſome others, by which the Moon's motions are ſo diſturbed, that hitherto by no law could they be reduced to any certain regulation. For the velocities or horary motions of the apogee and nodes of the Moon, and their equations, as well as the difference betwixt the greateſt eccentricity in the ſyzygies and the leaſt in the quadratures, and that inequality which we call the variation, in the progreſs of the year are augmented and diminiſhed (by *Cor.* 14, *Prop.* 66) in the triplicate ratio of the Sun's apparent diameter. Beſide that, the variation is mutable nearly in the duplicate ratio of the time between the quadratures (by *Cor.* 1 and 2, *Lemm.* 10) and *Cor.* 16, *Prop.* 66.) And all thoſe inequalities are ſomething greater in that part of the orbit which reſpects the Sun, than in the oppoſite part, but by a difference that is ſcarcely or not at all perceptible. [*]

P. 267.

And the diſtance of the Moon from the Earth to any given time.

BY a computation, which for brevities ſake I do not deſcribe, I alſo find that the area which the Moon by a radius drawn to the Earth, deſcribes in the ſeveral equal moments of time, is nearly as the ſum of the number $237\frac{3}{10}$ and verſed ſine of the double diſtance of the Moon from the neareſt quadrature in a circle whoſe radius is unity; and therefore that the ſquare of the Moon's diſtance from the Earth, is as that ſum divided by the horary motion of the Moon. Thus it is when the variation in the octants is in its mean quantity. But if the variation is greater or leſs, that verſed ſine muſt be augmented or diminiſhed in the ſame ratio. Let aſtronomers try how exactly the diſtances thus found will agree with the Moon's apparent diameters. [*]

FROM

P. 254.

The motions of the ſatellites of Jupiter and Saturn, derived from the motions of our Moon.

FROM the motions of our Moon we may derive the motions of the Moons or Satellites of Jupiter and Saturn. For the mean motion of the nodes of the outmoſt ſatellite of Jupiter is to the mean motion of the nodes of our Moon in a proportion compounded of the duplicate proportion of the periodic time of the Earth about the Sun, to the periodic time of Jupiter about the Sun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our Moon about the Earth (by *Cor.* 16, *Prop.* 66.) And therefore thoſe nodes, in the ſpace of an hundred years, are carried 8° 24′ backwards, or in *antecedentia*. The mean motions of the nodes of the inner ſatellites are to the (mean) motion of (the nodes of) the outmoſt as their periodic times to the periodic time of this, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite in *conſequentia* is to the motion of its nodes in *antecedentia*, as the motion of the apogee of our Moon, to the motion of its nodes (by the ſame corollary) and is thence given. The greateſt equations of the nodes and line of the apſes of each ſatellite are to the greateſt equations of the nodes and the line of the apſes of the Moon reſpectively, as the motion of the nodes and line of the apſes of the ſatellites in the time of one revolution of the firſt equations, to the motion of the nodes and apogæon of the Moon in the time of one revolution of the laſt equations. The variation of a ſatellite ſeen from Jupiter is to the variation of our Moon, in the ſame proportion as the whole motions of their nodes reſpectively, during the times in which the ſatellite and our Moon (after parting from) are revolved (again) to the Sun by the ſame corollary; and therefore in the outmoſt ſatellite the variation does not exceed 5″ 12‴. From the ſmall quantity of thoſe inequalities, and the ſlowneſs of the motions, it happens that the motions of the ſatellites are found to be ſo regular, that the more modern aſtronomers either deny all motion to the nodes, or affirm them to be very ſlowly regreſſive. [✳]

P. 238.

That the Planets, in reſpect of the fixed Stars, are revolved by equable motions about their proper axes. And that (perhaps) thoſe motions are the moſt fit for the equation of time.

WHILE the Planets are thus revolved in orbits about remote centers, in the mean time they make their ſeveral rotations about their proper axes; the Sun, in 26 days; Jupiter in $9^h\ 56'$; Mars, in $24\frac{2}{3}{}^h$; Venus, in 23^h; and that in plains not much inclined to the plain of the Ecliptic, and according to the order of the ſigns, as aſtronomers determine from the ſpots or maculæ, that by turns preſent themſelves to our ſight in their bodies. And there is a like revolution of our Earth performed in 24^h. And thoſe motions are neither accelerated nor retarded by the actions of the centripetal forces, as appears by *Cor.* 22, *Prop.* 66. And therefore of all others they are the moſt equable and moſt fit for the menſuration of time. But thoſe revolutions are to be reckoned equable, not from their return to the Sun, but to ſome fixed Star: For as the poſition of the Planets to the Sun is unequably varied, the revolutions of thoſe Planets from Sun to Sun are rendered unequable. [*]

Ibid.

That the Moon likewiſe is revolved by a diurnal motion about its axe, and that its libration does thence ariſe.

IN like manner is the Moon revolved about its axe by a motion moſt equable in reſpect of the fixed Stars, *viz.* in $27^d\ 7^h\ 43'$, that is, in the ſpace of a ſydereal month; ſo that this diurnal motion is equal to the mean motion of the Moon in its orbit. Upon which account, the ſame face of the Moon always reſpects the center about which this mean motion is performed, that is, the exteriour focus of the Moon's orbit nearly.

And hence ariſes a deflexion of the Moon's face from the Earth, ſometimes towards the eaſt, and other times towards the weſt, according to the poſition of the focus which it reſpects; and this deflexion is equal to the equation of the Moon's orbit, or to the difference betwixt its mean and true motions. And this is the Moon's libration in longitude. But it is likewiſe affected with a libration in latitude ariſing from the inclination of the Moon's axe to the plain of the orbit in which the Moon is revolved about the Earth. For that axe retains the ſame poſition to the fixed Stars nearly, and hence the poles preſent themſelves to our view by turns: As we may underſtand from the example of the motion of the Earth, whoſe poles, by reaſon of the inclination of its axe to the plain of the Ecliptic, are by turns illuminated by the Sun. To determine exactly the poſition of the Moon's axe to the fixed Stars, and the variation of this poſition, is a problem worthy of an aſtronomer. [*]

Of the preceſſion of the equinoxes, and of the libratory motion of the axes of the Earth and Planets.

By reaſon of the diurnal revolutions of the Planets, the matter which they contain endeavours to recede from the axes of this motion; and hence the fluid parts riſing

ſing higher towards the equator than about P. 239.
the poles, would lay the ſolid parts about
the equator under water, if thoſe parts
did not riſe alſo. Upon which account P. 239,
the Planets are ſomething thicker about 244, 245,
the equator than about the poles, and 251.
their equinoctial points thence become P. 252.
regreſſive, and their axes by a motion
of nutation, twice in every revolution
librate towards their ecliptics, and twice
return again to their former inclination,
as is explained in *Cor.* 18, *Prop.* 66.
And hence it is that Jupiter, viewed
through very long teleſcopes, does not
appear altogether round, but having its P. 244.
diameter that lies parallel to the Ecliptic,
ſomething longer than that which is drawn
from north to ſouth.

That the ſea ought twice to flow, and twice to ebb every day; that the higheſt water muſt fall out in the third hour after the appulſe of the luminaries to the meridian of the place.

AND from the diurnal motion and P. 255 to
the attractions of the Sun and Moon, 260.
our Sea ought twice to riſe and
twice to fall every day, as well
Lunar as Solar (by *Cor.* 19,
20, *Prop.* 66) and the greateſt
height of the water to happen
before the ſixth hour of
either day, and after the twelfth
hour preceding. By the ſlowneſs of
the diurnal motion, the flood is retracted
to the twelfth hour, and by the force
of the motion of reciprocation it is pro-

tracted and deferred till a time nearer to the ſixth hour. But till that time is more certainly determined by the phænomena, chooſing the middle between thoſe extremes, why may we not conjecture the greateſt height of the water to happen at the third hour? for thus the water will riſe all that time in which the force of the luminaries to raiſe it is greater, and will fall all that time in which their force is leſs; *viz.* from the ninth to the third hour when that force is greater, and from the third to the ninth when it is leſs. The hours I reckon from the appulſe of each luminary to the meridian of the place, as well under as above the horizon; and by the hours of the lunar day I underſtand the twenty-fourth parts of that time which the Moon ſpends before it comes about again by its apparent diurnal motion to the meridian of the place, which it left the day before.

That the greateſt tides happen in the ſyzygies of the luminaries, the leaſt in their quadratures: and that, at the third hour after the appulſe of the Moon to the meridian of the place. But that out of the ſyzygies and quadratures thoſe greateſt and leaſt tides deviate a little from that third hour towards the third hour after the appulſe of the Sun to the meridian.

But the two motions which the two luminaries raiſe will not appear diſtinguiſhed, but will make a certain mixt motion. In the conjunction or oppoſition of the luminaries, their forces will be conjoined and bring on the greateſt flood and ebb. In the quadratures the Sun will raiſe the waters which the Moon depreſſeth, and depreſs the waters which the Moon raiſeth, and from the difference of their forces, the ſmalleſt of all tides will follow. And becauſe (as experience tells us) the force of the Moon is greater than that of the Sun, the greateſt height of the water will happen about the third lunar hour. Out of the ſyzygies and quadratures the greateſt tide, which by the ſingle force of the Moon ought to fall out at the third lunar hour, and by the ſingle force of the Sun at the third ſolar hour, by the compound forces of both muſt fall out in an intermediate time that approaches nearer to the third hour of the Moon than to that of the Sun: And therefore while the Moon is paſſing from the ſyzygies to the quadratures, during which time the 3d hour of the Sun precedes the 3d of the Moon, the greateſt tide will precede the 3d lunar hour; and that by the greateſt interval a little after the octants of the Moon; and by like intervals, the greateſt tide will follow the 3d lunar hour, while the Moon is paſſing from the quadratures to the ſyzygies.

That the tides are greateſt when the luminaries are in their perigees.

But the effects of the luminaries depend upon their diſtances from the Earth. For when they are leſs diſtant, their effects are greater, and when more diſtant their effects are leſs, and that in the triplicate proportion of their apparent diameters. Therefore it is that the Sun in the winter time, being then in its perigee, has a greater effect, and makes the tides in the ſyzygies ſomething greater, and thoſe in the quadratures ſomething leſs, *cæteris paribus*, than in the ſummer ſeaſon; and every month the Moon, while in the perigee, raiſeth greater tides than at the diſtance of 15 days before or after, when it is in its apogee. Whence it comes to paſs that two higheſt tides do not follow one the other, in two immediately ſucceeding ſyzygies.

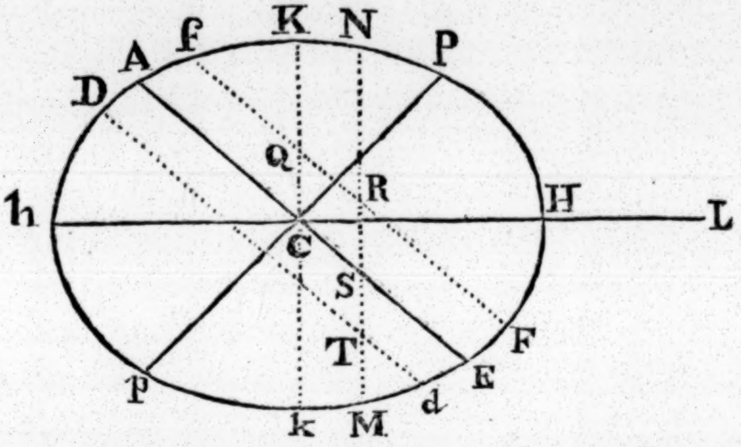
That the tides are greateſt about the equinoxes.

The effect of either luminary doth likewiſe depend upon its declination or diſtance from the equator. For if the luminary was placed at the pole, it would conſtantly attract all the parts of the waters, without any intenſion or remiſſion of its action, and could cauſe no reciprocation of motion. And therefore as the luminaries decline from the equator towards either pole, they will by degrees loſe their force, and on this account will excite leſſer tides in the ſolſtitial than in the equinoctial ſyzygies. But in the ſolſtitial quadratures, they will raiſe greater tides than in the quadratures about the equinoxes; becauſe the effect of the Moon, then ſituated in the equator, moſt exceeds the effect of the Sun. Therefore the greateſt tides fall out in thoſe ſyzygies, and the leaſt in thoſe quadratures, which happen about the time of both equinoxes. And the greateſt tide in the ſyzygies is always ſucceeded by the leaſt tide in the quadratures, as we find by experience. But becauſe the Sun is leſs diſtant from the Earth in winter than in ſummer, it comes to paſs that the greateſt and leaſt tides more frequently appear before than after the vernal equinox; and more frequently after, than before the autumnal.

That out of the equator the tides are greater and leſs alternately.

MOREOVER, the effects of the luminaries depend upon the latitudes of places. Let ApEP repreſent the Earth, on all ſides covered with deep waters; C its center; P, p its poles; AE the equator; F any place without the equator; Ff the parallel of the place; Dd the correſpondent parallel on the other ſide of the equator; L the place which the Moon

poſ-



poſſeſſed three hours before; H the place of the Earth directly under it; *h* the oppoſite place; K, *k* the places at 90 degrees diſtance; C H, C *h* the greateſt heights of the Sea from the center of the Earth, and C K, C *k*, the leaſt heights: and if with the axes H *h*, K *k*, an ellipſe is deſcribed, and by the revolution of that ellipſe about its longer axe H *h*, a ſpheroid H P K *h p k* is formed; this ſpheroid will nearly repreſent the figure of the Sea; and C F, C *f*, C D, C *d*, will repreſent the Sea in the places F, *f*, D, *d*. But further, if in the ſaid revolution of the ellipſe any point N deſcribes the circle N M, cutting the parellels F *f*, D *d* in any places R, T, and the equator A E in S; C N will repreſent the height of the Sea in all thoſe places, R, S, T, ſituated in this circle. Wherefore in the diurnal revolution of any place F, the greateſt flood will be in F, at the third Hour after the appulſe of the Moon to the meridian above the horizon; and afterwards the greateſt ebb in Q at the third hour after the ſetting of the Moon, and then the greateſt flood in *f*, at the third after the appulſe of the Moon to the meridian under the horizon, and laſtly the greateſt ebb in Q, at the third hour after the riſing of the Moon; and the latter flood in *f*, will be leſs than the preceding flood in F. For the whole Sea is divided into two huge and hemiſpherical floods, one in the hemiſphere K H *k* C, on the north ſide, the other in the oppoſite hemiſphere K H *k* C, which we may therefore call the northern and the ſouthern floods. Theſe floods being always oppoſite the one to the other, come by turns to the meridians of all places after the interval of twelve lunar hours. And ſeeing the northern countries partake more of the northern flood, and the ſouthern countries more of the ſouthern flood, thence ariſe tides alternately greater and leſs in all places without the equator, in which the luminaries riſe and ſet. But the greater tide will happen when the Moon declines towards the vertex of the place, about the third hour after the appulſe of the Moon to the meridian above the horizon; and when the Moon changes its declination, that which was the greater tide will be changed into a leſſer, and the greateſt difference of the floods will fall out about the times of the ſolſtices, eſpecially if the aſcending node of the Moon is about the firſt of Aries. So the morning tides in winter exceed thoſe of the evening, and the evening tides exceed thoſe of the morning in ſummer; at *Plymouth* by the height of one foot, but at *Briſtol* by the height of 15 inches, according to the obſervations of *Colepreſs* and *Sturmy*.

That by the conſervation of the impreſſed motion, the difference of the tides is diminiſhed: and that hence it may happen that the greateſt menſtrual tide will be the third after the ſyzygy.

BUT the motions which we have been deſcribing ſuffer ſome alteration from that force of reciprocation which the waters [having once received] retain a little while by their *vis inſita.* Whence it comes to paſs that the tides may continue for ſome time, though the actions of the luminaries ſhould ceaſe. This power of retaining the impreſſed motion leſſens the difference of the alternate tides, and makes thoſe tides which immediately ſucceed after the ſyzygies greater, and thoſe which follow next after the quadratures leſs. And hence it is that the alternate tides at *Plymouth* and *Briſtol*, do not differ much more one from the other, than by the height of a foot, or of 15 inches, and that the greateſt tides of all at thoſe ports are not the firſt but the third after the ſyzygies.

And beſides, all the motions are retarded in their paſſage through ſhallow channels, ſo that the greateſt tides of all in ſome ſtreights and mouths of rivers, are the fourth, or even the fifth after the ſyzygies. [*]

That the motions of the ſea may be retarded by impediments in its channels.

It may alſo happen that the greateſt tide may be the fourth or fifth after the ſyzygies, or fall out yet later, becauſe the motions of the Sea are retarded in paſſing through ſhallow places towards the ſhores. For ſo the tide arrives at the weſtern coaſt of *Ireland* at the third lunar hour, and an hour or two after, at the ports in the ſouthern coaſt of the ſame iſland, as alſo at the iſlands *Caſſiterides* commonly *Sorlings*; then ſucceſſively at *Falmouth*, *Plymouth*, *Portland*, the iſle of *Wight*, *Wincheſter*, *Dover*, the mouth of the *Thames*, and *London-Bridge*, ſpending twelve hours in this paſſage. But further the propagation of the tides may be obſtructed even by the channels of the ocean itſelf, when they are not of depth enough; for the flood happens at the third lunar hour in the *Canary* Iſlands, and at all thoſe weſtern coaſts that lie to-

towards the Atlantic ocean, as of *Ireland, France, Spain,* and all *Africa* to the cape of *Good-hope,* except in ſome ſhallow places, where it is impeded, and falls out later; and in the ſtreights of *Gibraltar,* where by reaſon of a motion propagated from the Mediterranean Sea it flows ſooner. But paſſing from thoſe coaſts over the breadth of the ocean to the coaſts of *America,* the flood arrives firſt at the moſt eaſtern ſhores of *Braſile,* about the fourth or fifth lunar hour, then at the mouth of the river of the *Amazons,* at the ſixth hour, but at the neighbouring iſlands, at the fourth hour; afterwards at the iſlands of *Bermudas* at the ſeventh hour, and at port St. *Auguſtin* in *Florida* at ſeven and a half. And therefore the tide is propagated through the ocean with a ſlower motion than it ſhould be according to the courſe of the Moon; and this retardation is very neceſſary, that the Sea at the ſame time may fall between *Braſile* and *New France,* and riſe at the *Canary* Iſlands, and on the coaſts of *Europe* and *Africa,* and *vice verſa:* For the Sea cannot riſe in one place but by falling in another. And it is probable that the *Pacific* Sea is agitated by the ſame laws. For in the coaſts of *Chili* and *Peru,* the higheſt flood is ſaid to happen at the third lunar hour. But with what velocity it is thence propagated to the eaſtern coaſts of *Japan*, the *Philippine* and other iſlands adjacent to *China*, I have not yet learned.

P. 260.

That from the impediments of channels and ſhores, various phænomena do ariſe, as that the ſea may flow but once every day.

FARTHER it may happen, that the tide may be propagated from the ocean through different channels towards the ſame port, and may paſs quicker through ſome channels than through others, in which caſe the ſame tide, divided into two or more ſucceeding one another, may compound new motions of different kinds. Let us ſuppoſe one tide to be divided into two equal tides; the former whereof precedes the other by the ſpace of ſix hours; and happens at the third or twenty-ſeventh hour from the appulſe of the Moon to the meridian of the port. If the Moon at the time of this appulſe to the meridian was in the equator, every ſix hours alternately there would ariſe equal floods, which meeting with as many equal ebbs, would ſo balance one the other, that for that day the water would ſtagnate and remain quiet. If the Moon then declined from the equator, the tides in the ocean would be alternately greater and leſs as was ſaid. And from hence two greater and two leſſer tides would be alternately propagated towards that port. But the two greater floods would make the greateſt height of the waters to fall out in the middle time betwixt both, and the greater and leſſer floods would make the waters to riſe to a mean height in the middle time between them, and in the middle time between the two leſſer floods the waters would riſe to their leaſt height. Thus in the ſpace of twenty-hours, the waters would come, not twice, but once only to their greateſt, and once only to their leaſt height; and their greateſt height, if the Moon declined towards the elevated pole, would happen at the ſixth or thirtieth hour after the appulſe of the Moon to the meridian, and when the Moon changed its declination this flood would be changed into an ebb.

Philoſoph. Tranſ. N° 162.

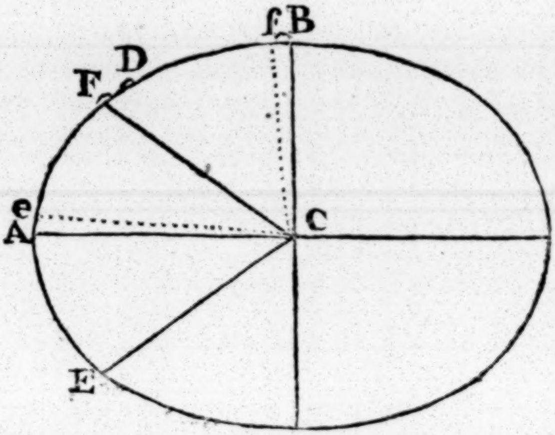
Of all which we have an example in the port of *Batſham*, in the kingdom of *Tunquin*, in the latitude of 20° 50′ north. In that port on the day which follows after the paſſage of the Moon over the equator the waters ſtagnate: When the Moon declines to the north, they begin to flow and ebb, not twice as in other ports, but once only every day, and the flood happens at the ſetting, and the greateſt ebb at the riſing of the Moon. This tide encreaſeth with the declination of the Moon, till the ſeventh or eighth day; then for the ſeventh or eighth day following, it decreaſeth at the ſame rate as it had increaſed before, and ceaſeth when the Moon changeth its declination. After which the flood is immediately changed into an ebb; and thenceforth the ebb happens at the ſetting, and the flood at the riſing of the Moon; till the Moon again changes its declination. There are two inlets from the ocean to this port; one more direct and ſhort between the iſland *Hainan* and the coaſt of *Quantung*, a province of *China*; the other round about between the ſame iſland and the coaſt of *Cochim*: And through the ſhorter paſſage the tide is ſooner propagated to *Batſham*. [*]

That the times of the tides within the channels of rivers are more unequal than in the ocean.

In the channels of rivers, the influx and reflux depends upon the current of the rivers, which obſtructs the ingreſs of the waters from the Sea, and promotes their egreſs to the Sea, making the ingreſs later and ſlower, and the egreſs ſooner and faſter. And hence it is, that the reflux is of longer duration than the influx, eſpecially far up the rivers, where the force of the Sea is leſs. So *Sturmy* tells us, that in the river *Avon* three miles below *Briſtol*, the water flows only five hours, but ebbs ſeven. And without doubt the difference is yet greater above *Briſtol*, as at *Careſham* or the *Bath*. This difference does likewiſe depend upon the quantity of the flux and reflux. For the more vehement motion of the Sea near the ſyzygies of the luminaries more eaſily overcoming the reſiſtance of the rivers, will make the ingreſs of the water to happen ſooner and to continue longer, and will therefore diminiſh this difference. But while the Moon is approaching to the ſyzygies, the rivers will be more plentifully filled, their currents being obſtructed by the greatneſs of the tides, and therefore will ſomething more retard the reflux of the Sea a little after, than a little before the ſyzygies. Upon which account the ſloweſt tides of all will not happen in the ſyzygies, but prevent them a little. And I obſerved above, that the tides before the ſyzygies were alſo retarded by the force of the Sun. And from both cauſes conjoined, the retardation of the tides will be both greater and ſooner before the ſyzygies. All which I find to be ſo, by the tide-tables which *Flamſteed* has compoſed from a great many obſervations.

That the tides are greater in greater and deeper Seas; greater on the ſhores of continents than of iſlands in the middle of the Sea; and yet greater in ſhallow bays that open with wide inlets to the Sea.

BY the laws we have been deſcribing, the times of the tides are governed: But the greatneſs of the tides depends upon the greatneſs of the Seas. Let C repreſent the center of the Earth, EADB the oval figure of the Sea, CA the longer ſemiaxe of this oval, CB the ſhorter inſiſting at right angles upon the former, D the middle point between A and B, and ECF or eCf the angle at the center of the Earth, ſubtended by the breadth of the Sea that terminates in the ſhores E, F, or e, f. Now ſuppoſing that the point A is in the middle between the points E, F, and the point D in the middle between the points e, f; if the difference of the heights CA, CB repreſent the quantity of the tide in a very deep Sea ſur-

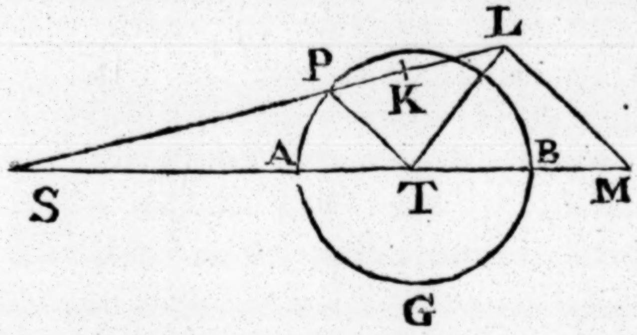


ſurrounding the whole Earth; the exceſs of the height CA above the height CE or CF, will repreſent the quantity of the tide in the middle of the Sea EF terminated by the ſhores E, F: and the exceſs of the height C *e* above the height C *f*, will nearly repreſent the quantity of the tide on the ſhores, *f*, of the ſame Sea. Whence it appears that the tides are far leſs in the middle of the ſea, than at the ſhores; and that the tides at the ſhores are nearly as EF the breadth of the Sea, P. 309. not exceeding a quadrantal arc. And hence it is that near the equator, where the Sea between *Africa* and *America* is narrow, the tides are far leſs than towards either ſide in the temperate Zones, where the Seas are extended wider, or on almoſt all the ſhores of the *Pacific* Sea, as well towards *America* as towards *China*, and within as well as without the tropicks; and that in iſlands in the middle of the ſea they ſcarce riſe higher than two or three feet, but on the ſhores of great continents are three or four times greater and above, eſpecially if the motions propagated from the ocean are by degrees contracted into a narrow ſpace, and the water, to fill and empty the bays alternately, is forced to flow and ebb with great violence through ſhallow places; as *Plymouth* and *Chepſtow*-

Bridge in *England*, at the mount of *St. Michael* and town of *Avranches* in *Normandy*, and at *Cambaja* and *Pegu* in the *Eaſt Indies*. In which places, the ſea hurried in and out with great violence, ſometimes lays the ſhores under water, ſometimes leaves them dry for many miles. Nor is the force of the influx and efflux to be broke till it has raiſed or depreſſed the water to forty or fifty feet and more. Thus alſo long and ſhallow ſtreights that open to the ſea with mouths wider and deeper than the reſt of their channel, (ſuch as thoſe about *Britain*, and the *Magellanic Streights* at the eaſtern entry) will have a greater flood and ebb, or will more intend and remit their courſe, and therefore will riſe higher and be depreſſed lower. On the coaſts of *South America*, it is ſaid that the *Pacific* Sea in its reflux ſometimes retreats two miles, and gets out of ſight of thoſe that ſtand on ſhore. Whence in theſe places, the floods will be alſo higher. But in deeper waters the velocity of influx and efflux is always leſs, and therefore the aſcent and deſcent is ſo too. Nor in ſuch places is the ocean known to aſcend to more than ſix, eight, or ten feet. The quantity of the aſcent I compute in the following manner. [*]

P. 262, 263.

Let S repreſent the Sun, T the Earth, P the Moon, PAGB the Moon's orbit.



The force of the Sun to diſturb the motions of the Moon, computed from the foregoing principles.

In SP take SK equal to ST and SL to SK in the duplicate ratio of SK to SP. Parallel to PT draw LM; and ſuppoſing the mean quantity of the circum-ſolar force directed towards the Earth to be repreſented by the diſtance ST or SK, SL will repreſent the quantity thereof directed towards the Moon. But that force is compounded of the parts SM, LM; of which the force LM, and that part of SM which is repreſented by TM do diſturb the motion of the Moon (as appears from *Prop.* 66, and its *Corollaries.*) In ſo far as the Earth and Moon are revolved about their common center of gravity, the Earth will be liable to the action of the like forces. But we may refer the ſums as well of the forces as of the motions to the Moon, and repreſent the ſums of the forces by the lines TM and ML, which are proportional to them. The force LM, in its mean quantity, is to the force by which the Moon may be revolved in an orbit, about the Earth quieſcent, at the diſtance PT, in the duplicate ratio of the Moon's periodic time about the Earth to the Earth's periodic time about the Sun (by *Cor.* 17, *Prop.* 66) that is, in the duplicate ratio of $27^d\ 7^h\ 43'$ to $365^d\ 6^h\ 9'$, or as 1000 to 178725, or 1 to $178\frac{29}{40}$. The force by which the Moon may be revolved in its orb about the Earth in reſt, at the diſtance PT of $60\frac{1}{2}$ ſemi-diameters of the Earth, is to the force by which it may revolve in the ſame time at the diſtance of 60 ſemi-diameters as $60\frac{1}{2}$ to 60; and this force is to the force of gravity with us as 1 to 60×60 nearly. And therefore the mean force ML is to the force of gravity at the ſurface of the Earth as $1 \times 60\frac{1}{2}$ to $60 \times 60 \times 60 \times 178\frac{29}{40}$, or 1 to 638092,6. Whence the force TM will be alſo given from the proportion of the lines TM, ML. And theſe are the forces of the Sun, by which the Moon's motions are diſturbed.

P. 305.

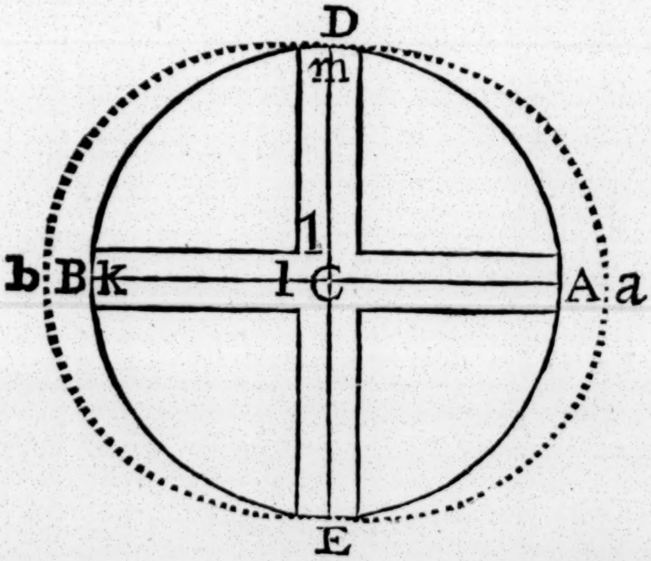
The force of the Sun to move the ſea computed.

If from the Moon's orbit we deſcend to the Earth's ſurface, thoſe forces will be diminiſhed in the ratio of the diſtances $60\frac{1}{2}$ and 1;

and therefore the force LM will then become 38604600 times leſs than the force of gravity. But this force acting equally every where upon the Earth, will ſcarcely effect any change on the motion of the Sea, and therefore may be neglected in the explication of that motion. The other force TM, in places where the Sun is vertical or in their nadir, is triple the quantity of the force ML, and therefore but 12868200 times leſs than the force of gravity. [✳]

The height of the tide under the equator, ariſing from the force of the Sun, computed.

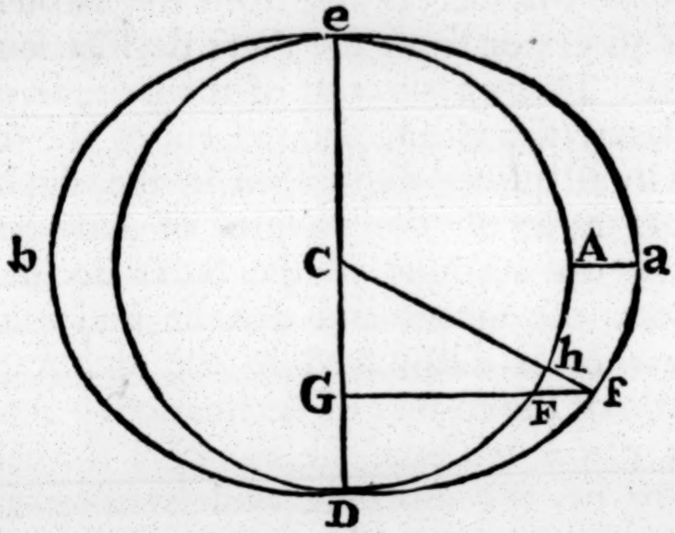
SUPPOSE now ADBE to repreſent the ſpherical ſurface of the Earth, *a*D*b*E the ſurface of the water over-ſpreading it, C the center of both, A the place to which the Sun is vertical, B the place



oppoſite; D, E, places at 90 degrees diſtance from the former; A C E *m l k* a right angled cylindric canal paſſing through the Earth's center. The force TM in any place is as the diſtance of the place from the plain DE, on which a line from A to C inſiſts at right angles, and therefore in the part of the canal which is repreſented by E C *lm*, is of no quantity; but in the other part A C *lk*, is as the gravity at the ſeveral heights. For in deſcending towards the center of the Earth, gravity is (by *Prop.* 73.) every where as the height. And therefore the force T M drawing the water upwards will diminiſh its gravity in the leg A C *l k* of the canal in a given ratio; upon which account the water will aſcend in this leg, till its defect of gravity is ſupplied by its greater height, nor will it reſt in an equilibrium, till its total gravity becomes equal to the total gravity in E C *l m* the other leg of the canal. Becauſe the gravity of every particle is as its diſtance from the Earth's center, the weight of the whole water in either leg will increaſe in the duplicate ratio of the height; and therefore the height of the water in the leg A C *l k* will be to theh eight thereof in the leg C *l m* E in the ſubduplicate ratio of the number 12868201 to 12868200, or in the ratio of the number 25623053 to the number 25623052, and the height of the water in the leg E C *l m* to the difference of the heights, as 25623052 to 1. But the height in the leg E C *l m* is of 19615800 *Paris* feet, as has been lately found by the menſuration of the *French*. And therefore by the preceding analogy, the difference of the heights comes out $9\frac{1}{5}$ inches of the *Paris* foot; and the Sun's force will make the height of the Sea at A to exceed the height of the ſame at E by 9 inches. And though the water of the canal A C E *m l k* be ſuppoſed to be frozen into a hard and ſolid conſiſtence, yet the heights thereof at A and E, and all other intermediate places, would ſtill remain the ſame.

The height of the tides under the parallels, ariſing from the Sun's force, computed.

LET A *a* (in the following figure) repreſent that exceſs of height of nine inches at A, and *h f* the exceſs of height at any other place *h*; and upon DC let fall the perpendicular *f* G, meeting the globe of the Earth in F. And becauſe the diſtance of the Sun is ſo great that all the right lines drawn thereto may be conſidered as parallel, the force TM in any place *f*, will be to the ſame force in the place A, as the ſine FG to the radius AC. And therefore ſince thoſe forces tend to the Sun in the direction of



of parallel lines, they will generate the parallel heights F*f*, A*a* in the ſame ratio; and therefore the figure of the water D*faeb* will be a ſpheroid made by the revolution of an ellipſe about its longer axe *ab*. And the perpendicular height *fh* will be to the oblique height F*f* as *f* G to *f* C, or as F G to A C: and therefore the height *fh* is to the height A*a* in the duplicate ratio of F G to A C, that is, in the ratio of the verſed ſine of double the angle D C*f* to double the radius, and is thence given. And hence to the ſeveral moments of the apparent revolution of the Sun about the Earth, we may infer the proportion of the aſcent and deſcent of the waters at any given place under the equator, as well as of the diminution of that aſcent and deſcent, whether ariſing from the latitude of places or from the Sun's declination; *viz.* That on account of the latitude of places, the aſcent and deſcent of the ſea is in all places diminiſhed in the duplicate ratio of the co-ſines of latitude; and on account of the Sun's declination, the aſcent and deſcent under the equator is diminiſhed in the duplicate ratio of the co-ſine of declination. And in places without the equator, the half ſum of the morning and evening aſcents (that is, the mean aſcent) is diminiſhed nearly in the ſame ratio.

The proportion of the tides under the equator, in the ſyzygies and quadratures, ariſing from the joint forces of both Sun and Moon.

LET S and L reſpectively repreſent the forces of the Sun and Moon placed in the equator, and at their mean diſtances from the Earth, R the radius, T and V the verſed ſines of double the complements of the Sun and Moon's declinations to any given time, D and E the mean apparent diameters of the Sun and Moon: And, ſuppoſing F and G to be their apparent diameters to that given time, their forces to raiſe the tides under the equator will be, in the ſyzygies, $\frac{VG^3}{2RE^3}L + \frac{TF^3}{2RD^3}S$, in the quadratures, $\frac{VG^3}{2RE^3}L - \frac{TF^3}{2RD^3}S$. And if the ſame ratio is likewiſe obſerved under the parallels, from obſervations accurately made in our northern climates we may determine the proportion of the forces L and S; and then by means of this rule predict the quantities of the tides to every ſyzygy and quadrature.

P. 306, 307, 308, 309.
The force of the Moon to excite tides, and the height of the water thence ariſing, computed.

AT the mouth of the river *Avon*, three miles below *Briſtol*, in ſpring and autumn, the whole aſcent of the water in the conjunction or oppoſition of the luminaries (by the obſervation of *Sturmy*) is about 45 feet, but in the quadratures only 25. Becauſe the apparent diameters of the luminaries are not here determined, let us aſſume them in their mean quantities, as well as the Moon's declination in the equinoctial quadratures, in its mean quantity, that is, $23\frac{1}{2}°$; and the verſed ſine of double its complement will be 1682, ſuppoſing the radius to be 1000. But the declinations of the Sun in the equinoxes and of the Moon in the ſyzygies are of no quantity, and the verſed ſines of double the complements are each 2000. Whence thoſe forces become L + S in the ſyzygies, and $\frac{1682}{2000}L - S$ in the quadratures,

re-

reſpectively proportional to the heights of the tides of 45 and 25 feet, or of 9 and 5 paces. And therefore multiplying the extremes and the means we have $5\,L + 5\,S = \frac{15138}{2000}L - 9\,S$, or $L = \frac{28000}{5138}S = 5\frac{5}{11}\,S$.

But further, I remember to have been told, that in ſummer the aſcent of the ſea in the ſyzygies, is to the aſcent thereof in the quadratures as about 5 to 4. In the ſolſtices themſelves it is probable that the proportion may be ſomething leſs, as about 6 to 5; whence it would follow that L is $= 5\frac{1}{6}\,S$. [For then the proportion is $\frac{1682}{2000}L + \frac{1682}{2000}S : L - \frac{1682}{2000}S :: 6 : 5$.] Till we can more certainly determine the proportion from obſervation, let us aſſume $L = 5\frac{1}{3}\,S$; and ſince the heights of the tides are as the forces which excite them, and the force of the Sun is able to raiſe the tides to the height of nine inches, the Moon's force will be ſufficient to raiſe the ſame to the height of four feet. And if we allow that this height may be doubled, or perhaps tripled by that force of reciprocation which we obſerve in the motion of the waters, and by which their motion once begun is kept up for ſome time, there will be force enough to generate all that quantity of tides, which we really find in the ocean.

THUS we have ſeen that theſe forces are ſufficient to move the ſea. But, ſo far as I can obſerve, they will not be able to produce any other effect ſenſible on our Earth. For ſince the weight of one grain in 4000 is not ſenſible in the niceſt balance; and the Sun's force to move the tides is 12868200 leſs than the force of gravity; and the ſum of the forces of both Moon and Sun, exceeding the Sun's force only in the ratio of $6\frac{1}{3}$ to 1, is ſtill 2032890 times leſs than the force of gravity; it is evident that both forces together are 500 times leſs than what is required ſenſibly to increaſe or diminiſh the weight of any body in a balance. And therefore they will not ſenſibly move any ſuſpended body; nor will they produce any ſenſible effect on pendulums, barometers, bodies ſwimming in ſtagnant water, or in the like ſtatical experiments. In the atmoſphere indeed they will excite ſuch a flux and reflux as they do in the ſea, but with ſo ſmall a motion that no ſenſible wind will be thence produced. [*]

P. 310.

That thoſe forces of the Sun and Moon are ſcarce ſenſible by any other effect beſide the tides which they raiſe in the ſea.

P. 311.

That the body of the Moon is about 6 times more denſe than the body of the Sun.

If the effects of both Moon and Sun in raiſing the tides, as well as their apparent diameters, were equal among themſelves, their abſolute forces would (by *Cor.* 14. *Prop.* 66.) be as their magnitudes. But the effect of the Moon is to the effect of the Sun as about $5\frac{1}{3}$ to 1; and the Moon's diameter leſs than the Sun's in the ratio of $31\frac{1}{2}$ to $32\frac{1}{5}$ or of 45 to 46. Now the force of the Moon is to be increaſed in the ratio of the effect directly, and in the triplicate ratio of the diameter inverſely. Whence the force of the Moon compar'd with its magnitude will be to the force of the Sun compar'd with its magnitude in the ratio compounded of $5\frac{1}{3}$ to 1, and the triplicate of 45 to 46 inverſely, that is, in the ratio of about $5\frac{7}{10}$ to 1. And therefore the Moon in reſpect of the magnitude of its body, has an abſolute centripetal force greater than the Sun in reſpect of the magnitude of its body, in the ratio of $5\frac{7}{10}$ to 1, and is therefore more denſe in the ſame ratio.

That the Moon is more denſe than the Earth in a ratio of about 3 to 2.

In the time of $27^{d}\ 7^{h}\ 43'$ in which the Moon makes its revolution about the Earth, a Planet may be revolved about the Sun at the diſtance of 18,954 diameters of the Sun from the Sun's center, ſuppoſing the mean apparent diameter of the Sun to be $32\frac{1}{5}'$. And in the ſame time the Moon may be revolved about the Earth at reſt, at the diſtance of 30 of the Earth's diameters. If in both caſes the number of diameters was the ſame, the abſolute circum-terreſtrial force would (by *Cor.* 2. *Prop.*72) be to the abſolute circum-ſolar force as the magnitude of the Earth to the magnitude of the Sun. Becauſe the number of the Earth's diameters is greater in the ratio of 30 to 18,954, the body of the Earth will be leſs in the triplicate of that ratio, that is, in the ratio of $3\frac{28}{29}$ to 1. Wherefore the Earth's force, for the magnitude of its body, is to the Sun's force, for the magnitude of its body, as $3\frac{28}{29}$ to 1, and conſequently the Earth's denſity to the Sun's will be in the ſame ratio. Since then the Moon's denſity is to the Sun's denſity as $5\frac{7}{10}$ to 1, the Moon's denſity will be to the Earth's denſity as $5\frac{7}{10}$ to $3\frac{28}{29}$, or as 23 to 16. Wherefore ſince the Moon's magnitude is to the Earth's magnitude as about 1 to $41\frac{1}{2}$, the Moon's abſolute centripetal force will be to the Earth's abſolute centripetal force as about 1 to 29, and the quantity of matter in the Moon to the quantity of matter in the Earth in the ſame ratio. And hence the common center of gravity of the Earth and Moon is more exactly determined [*] than hitherto has been done. From the knowledge of which we may now infer the Moon's diſtance from the Earth with greater accuracy. But I would rather wait till the proportion of the bodies of the Moon and Earth one to the other is more exactly defined from the phænomena of the tides, hoping that in the mean time the circumference of the Earth may be meaſured from more diſtant ſtations than any body has yet employed for this purpoſe.

Of the diſtance of the fixed Stars.

THUS I have given an account of the ſyſtem of the Planets. As to the fixed Stars, the ſmallneſs of their annual parallax proves them to be removed to immenſe diſtances from the ſyſtem of the Planets. That this parallax is leſs than one minute is moſt certain; and from thence it follows that the diſtance of the fixed Stars is above 360 times greater than the diſtance of Saturn from the Sun. Such as reckon the Earth one of the Planets and the Sun one of the fixed Stars, may remove the fixed Stars to yet greater diſtances by the following arguments. From the annual motion of the Earth there would happen an apparent tranſpoſition of the fixed Stars, one in reſpect of another, almoſt equal to their double parallax. But the greater and nearer Stars, in reſpect of the more remote, which are only ſeen by the teleſcope, have not hitherto been obſerved to have the leaſt motion. If we ſhould ſuppoſe that motion to be but leſs than $20''$, the diſtance of the nearer fixed Stars would exceed the mean diſtance of Saturn by above 2000 times. Again, the diſc of Saturn, which is only $17''$ or $18''$ in diameter, receives but about $\frac{1}{2100000000}$ of the Sun's light. For ſo much leſs is that diſc than the whole ſpherical ſurface of the orb of Saturn. Now if we ſuppoſe Saturn to reflect about $\frac{1}{4}$ of this light, the whole light reflected from its illuminated hemiſphere will be about $\frac{1}{4200000000}$ of the whole light emitted from the Sun's hemiſphere. And therefore ſince light is rarefied in the duplicate ratio of the diſtance from the luminous body, if the Sun was $10000\sqrt{42}$ times more diſtant than Saturn, it would yet appear as lucid, as Saturn now does without its ring, that is, ſomething more lucid than a fixed Star of the firſt magnitude. Let us therefore ſuppoſe that the diſtance from which the Sun would ſhine as a fixed Star exceeds that of Saturn by about 100,000 times, and its apparent diameter will be $7^{v}\ 16^{vi}$, and its parallax ariſing from the annual motion of the Earth $13''''$. And ſo great will be the diſtance, the apparent diameter, and the parallax of the fixed Stars of the firſt magnitude in bulk and light equal to our Sun. Some may perhaps imagine that a great part of the light of the fixed Stars is intercepted and loſt in its paſſage through ſo vaſt ſpaces, and upon that account pretend to place the fixed Stars at nearer diſtances. But at this rate the remoter Stars could be ſcarce ſeen. Suppoſe for example that $\frac{3}{4}$ of the light periſh in its paſſage from the neareſt fixed Stars to us; then $\frac{3}{4}$ will twice periſh in its paſſage through a double ſpace, thrice through a triple, and ſo forth. And therefore the fixed Stars that are at a double diſtance will be 16 times more obſcure, *viz.* 4 times more obſcure on account of the diminiſhed apparent diameter; and again, 4 times more on account of the loſt light. And by the ſame argument, the fixed Stars at a triple diſtance, will be $9\times4\times4$ or 144 times more obſcure, and thoſe at a quadruple diſtance will be $16\times4\times4\times4$ or 1024 times more obſcure. But ſo great a diminution of light is no ways conſiſtent with the phænomena and with that hypotheſis which places the fixed Stars at different diſtances.

P. 323, 324.

That the Comets as often as they become viſible to us, are nearer than Jupiter. Proved from their parallax in longitude.

THE fixed Stars being therefore at ſuch vaſt diſtances from one another, can neither attract each other ſenſibly, nor be attracted by our Sun. But the Comets muſt unavoidably be acted on by the circumſolar force. For as the Comets were placed by aſtronomers above the Moon, becauſe they were found to have no diurnal parallax; ſo their annual parallax is a convincing proof of their deſcending into the regions of the Planets. For all the Comets which move in a direct courſe, according to the order of the ſigns, about the end of their appearance become more than ordinarily ſlow, or retrograde, if the Earth is between them and the Sun; and more than ordinarily ſwift, if the Earth is approaching to a heliocentric oppoſition with them. Whereas on the other hand, thoſe which move againſt the order of the ſigns, towards the end of their appearance, appear ſwifter than they ought to be if the Earth is between them and the Sun; and ſlower, and perhaps retrograde, if the Earth is in the other ſide of its orbit. This is occaſioned by the motion of the Earth in different ſituations.

ons. If the Earth go the ſame way with the Comet, with a ſwifter motion, the Comet becomes retrograde; if with a ſlower motion, the Comet becomes ſlower however; and if the Earth move the contrary way, it becomes ſwifter. [*] And by collecting the differences between the ſlower and ſwifter motions, and the ſums of the more ſwift and retrograde motions, and comparing them with the ſituation and motion of the Earth from whence they ariſe, I found, by means of this parallax, that the diſtances of the Comets at the time they ceaſe to be viſible to the naked eye, is always leſs than the diſtance of Saturn; and generally even leſs than the diſtance of Jupiter.

P. 325.

The ſame proved from their parallax in latitude.

THE ſame thing may be collected from the curvature of the way of the Comets. Theſe bodies go on nearly in great circles while their motion continues ſwift; but about the end of their courſe, when that part of their apparent motion, which ariſes from the parallax, bears a greater proportion to their whole apparent motion, they commonly deviate from thoſe circles; and when the Earth goes to one ſide, they deviate to the other. And this deflexion, becauſe of its correſponding with the motion of the Earth, muſt ariſe chiefly from the parallax; and the quantity thereof is ſo conſiderable, as by my computation, to place the diſappearing Comets a good deal lower than Jupiter. Whence it follows, that when they approach nearer to us in their perigees and perihelions, they often deſcend below the orbits of Mars and the inferiour Planets. [*]

The ſame proved otherwiſe by the parallax.

MOREOVER, this nearneſs of the Comets is confirmed by the annual parallax of the orbit, in ſo far as the ſame is pretty nearly collected by the ſuppoſition that the Comets move uniformly in right lines. The method of collecting the diſtance of a Comet according to this hypotheſis from four obſervations, (firſt attempted by *Kepler*, and perfected by Dr. *Wallis* and Sir *Chriſtopher Wren*) is well known. And the Comets reduced to this regularity generally paſs through the middle of the planetary region. So the Comets of the years 1607 and 1618, as their motions are defined by *Kepler*, paſſed between the Sun and the Earth; that of the year 1664 below the orbit of Mars; and that in 1680 below the orbit of Mercury, as its motion was defined by Sir *Chriſtopher Wren* and others. By a like rectilinear hypotheſis, *Hevelius* placed all the Comets about which we have any obſervations, below the orbit of Jupiter. 'Tis a falſe notion therefore, and contrary to aſtronomical calculation, which ſome have entertained, who from the regular motion of the Comets, either remove them into the regions of the fixed Stars, or deny the motion of the Earth; whereas their motions cannot be reduced to perfect regularity, unleſs we ſuppoſe them to paſs through the regions near the Earth in motion. And theſe are the arguments drawn from the parallax, ſo far as it can be determined without an exact knowledge of the orbits and motions of the Comets.

P. 326, to 329.

From the light of the Comets heads it is proved that they deſcend to the orbit of Saturn.

THE near approach of the Comets is further confirmed from the light of their heads. For the light of a celeſtial body, illuminated by the Sun, and receding to remote parts, is diminiſhed in the quadruplicate proportion of the diſtance, to wit, in one duplicate proportion on account of the increaſe of the diſtance from the Sun; and in another duplicate proportion, on account of the decreaſe of the apparent diameter. Hence it may be inferred, that Saturn being at a double diſtance, and having its apparent diameter nearly half of that of Jupiter, muſt appear about 16 times more obſcure;

and that if its diſtance were 4 times greater, its light would be 256 times leſs; and therefore would be hardly perceivable to the naked eye. But now the Comets often equal Saturn's light, without exceeding him in their apparent diameters. So the Comet of the year 1668, according to Dr. *Hook*'s obſervations, equalled in brightneſs the light of a fixed Star of the firſt magnitude, and its head, or the Star in the middle of the coma, appeared, through a teleſcope of 15 foot, as lucid as Saturn near the horizon. But the diameter of the head was only 25″; that is, almoſt the ſame with the diameter of a circle equal to Saturn and his ring. The coma or hair ſurrounding the head was about ten times as broad; namely, $4\frac{1}{6}$ min. Again, the leaſt diameter of the hair of the Comet of the year 1682, obſerved by Mr. *Flamſteed* with a tube of 16 foot, and meaſured with the micrometer, was 2′ 0″. But the nucleus, or ſtar in the middle, ſcarce poſſeſſed the tenth part of this breadth, and was therefore only 11″ or 12″ broad. But the light and clearneſs of its head exceeded that of the year 1680, and was equal to that of the Stars of the firſt or ſecond magnitude. Moreover, the Comet of the year 1665 in *April*, as *Hevelius* informs us, exceeded almoſt all the fixed Stars in ſplendor; and even Saturn it ſelf, as being of a much more vivid colour. For this Comet was more lucid than that which appeared at the end of the foregoing year, and was compared to the Stars of the firſt magnitude. The diameter of the coma was about 6′; but the nucleus, compared with the Planets by means of a teleſcope, was plainly leſs than Jupiter, and was ſometimes thought leſs, ſometimes equal to the body of Saturn within the ring. To this breadth add that of the ring, and the whole face of Saturn will be twice as great as that of the Comet, with a light not at all more intenſe: and therefore the Comet was nearer to the Sun than Saturn. From the proportion of the nucleus to the whole head found by theſe obſervations, and from its breadth, which ſeldom exceeds 8′ or 12′, it appears that the Stars of the Comets are moſt commonly of the ſame apparent magnitude as the Planets; but that their light may be compared oftentimes with that of Saturn, and ſometimes exceeds it. And hence 'tis certain that in their perihelia their diſtances can ſcarce be greater than that of Saturn. At twice that diſtance, the light would be four times leſs, which beſides by its dim paleneſs would be as much inferiour to the light of Saturn, as the light of Saturn is to the ſplendour of Jupiter: But this difference would be eaſily obſerved. At a diſtance ten times greater their bodies muſt be greater than that of the Sun; but their light would be 100 times fainter than that of Saturn. And at diſtances ſtill greater their bodies would far exceed the Sun: but being in ſuch dark regions, they muſt be no longer viſible. So impoſſible is it to place the Comets in the middle regions between the Sun and fixed Stars, accounting the Sun as one of the fixed Stars. For certainly they would receive no more light there from the Sun, than we do from the greateſt of the fixed Stars.

And alſo below the orb of Jupiter, and ſometimes below the orb of the Earth.

So far we have gone without conſidering that obſcuration which Comets ſuffer from that plenty of thick ſmoak which encompaſſeth their heads, and through which the heads always ſhew dull as through a cloud. For by how much the more a body is obſcured by this ſmoak, by ſo much the more near it muſt be allowed to come to the Sun, that it may vie with the Planets in the quantity of light which it reflects. Whence it is probable that the Comets deſcend far below the orbit of Saturn, as we proved before from their parallax. But above all, the thing is evinced from their tails, which muſt be owing either to the Sun's light reflected from a ſmoak ariſing from them and diſperſing it ſelf through the Æther, or to the light of their own heads.

In the former caſe we muſt ſhorten the diſtance of the Comets, leſt we be obliged to allow that the ſmoak ariſing from their heads is propagated through ſuch a vaſt extent of ſpace, and with ſuch a velocity of expanſion as will ſeem altogether incredible. In the latter caſe the whole light of both head and tail muſt be aſcribed to the central nucleus. But then if we ſuppoſe all this light to be united and condenſed within the diſc of the nucleus, certainly the nucleus will by far exceed Jupiter it ſelf in ſplendour, eſpecially when it emits a very large and lucid tail. If therefore under a leſs apparent diameter it reflects more light, it muſt be much [*] more illuminated by the Sun, and therefore much nearer to it. So the Comet that appeared *Dec.* 12. and 15. *O. S. Anno* 1679, at the time it emitted a very ſhining tail, whoſe ſplendour was equal to that of many Stars like Jupiter, if their light were dilated and ſpread through ſo great a ſpace, was, as to the magnitude of its nucleus, leſs than Jupiter, (as Mr. *Flamſteed* obſerved) and therefore was much nearer to the Sun: Nay, it was even leſs than Mercury. For on the 17th of that month when it was nearer to the Earth, it appeared to *Caſſini* through a teleſcope of 35 foot a little leſs than the globe of Saturn. On the 8th of this month, in the morning, Dr. *Halley* ſaw the tail, appearing broad and very ſhort, and as if it roſe from the body of the Sun it ſelf, at that time very near its riſing. Its form was like that of an extraordinary bright cloud; nor did it diſappear till the Sun it ſelf began to be ſeen above the horizon. Its ſplendour therefore exceeded the light of the clouds till the Sun roſe, and far ſurpaſſed that of all the Stars together, as yielding only to the immediate brightneſs of the Sun it ſelf. Neither Mercury, nor Venus, nor the Moon it ſelf are ſeen ſo near the riſing Sun. Imagine all this dilated light collected together, and to be crouded into the orbit of the Comet's nucleus which was leſs than Mercury; by its ſplendour thus increaſed, becoming ſo much more conſpicuous, it will vaſtly exceed Mercury, and therefore muſt be nearer to the Sun. On the 12th and 15th of the ſame month this tail extending it ſelf over a much greater ſpace, appeared more rare; but its light was ſtill ſo vigorous as to become viſible when the fixed Stars were hardly to be ſeen, and ſoon after to appear like a fiery beam ſhining in a wonderful manner. From its length, which was 40 or 50 degrees, and its breadth of 2 degrees, we may compute what the light of the whole muſt be.

The ſame proved from the extraordinary ſplendour of their tails when they are near the Sun.

THIS near approach of the Comets to the Sun is confirmed from the ſituation they are ſeen in when their tails appear moſt reſplendent. For when the head paſſes by the Sun and lies hid under the ſolar rays, very bright and ſhining tails, like fiery beams, are ſaid to iſſue from the horizon; but afterwards when the head begins to appear, and is got farther from the Sun, that ſplendour always decreaſes and turns by degrees into a paleneſs like to that of the milky way, but much more ſenſible at firſt; after that vaniſhing gradually. Such was that moſt reſplendent Comet deſcribed by *Ariſtotle*, *Lib.* 1. *Meteor.* 6. 'The 'head thereof could not be ſeen becauſe 'it ſet before the Sun, or at leaſt was hid 'under the Sun's rays; but the next day 'it was ſeen as well as might be; for having left the Sun but a very little way, it 'ſet

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'ſet immediately after it. And the ſcattered light of the head obſcured by the 'too great ſplendour (of the tail) did not 'yet appear. But afterwards (ſays *Ariſtotle*) when the ſplendour of the tail 'was now diminiſhed, (the head of) the 'Comet recovered its native brightneſs. 'And the ſplendour of its tail reached 'now to a third part of the heavens' (that is to 60°.) 'It appeared in the winter 'ſeaſon, and riſing to *Orion*'s Girdle, [*] 'there vaniſhed away.' Two Comets of the ſame kind are deſcribed by *Juſtin, lib.* 37. which according to his account 'ſhined ſo bright that the whole heaven 'ſeemed to be on fire; and by their greatneſs filled up a fourth part of the heavens, and by their ſplendour exceeded 'that of the Sun.' By which laſt words a near poſition of theſe bright Comets and the riſing or ſetting Sun is intimated. *P.* 373, 374. We may add to theſe the Comet of the year 1101 or 1106, 'the ſtar of which 'was ſmall and obſcure (like that of '1680) but the ſplendour ariſing from it 'extremely bright, reaching like a fiery 'beam to the Eaſt and North,' as *Hevelius* has it from *Simeon* the monk of *Durham*. It appeared at the beginning of *February* about the evening in the ſouth-weſt. From this and from the ſituation of the tail we may infer that the head was near the Sun. *Matthew Paris* ſays, ‘ it was about one cubit from the ‘ Sun; from the third’ [or rather the ſixth] ‘ to the ninth hour ſending out a ‘ long ſtream of light.’ The Comet of [*] 1264 in *July*, or about the ſolſtice, preceded the riſing Sun, ſending out its beams with a great light towards the weſt as far as the middle of the heavens. And at the beginning it aſcended a little above the horizon; but as the Sun went forwards it retired every day farther from the horizon, till it paſſed by the very middle of the heavens. It is ſaid to have been at the beginning large and bright, having a large coma, which decayed from day to day. It is deſcribed in *Append. Matth. Pariſ. Hiſt. Ang.* after this manner. *An. Chriſti* 1265, ‘ there appeared a Comet ſo wonderful, ‘ that none then living had ever ſeen the ‘ like. For riſing from the eaſt with a ‘ great brightneſs, it extended it ſelf with ‘ a great light as far as the middle of the ‘ hemiſphere towards the weſt.’ The Latin original being ſomewhat barbarous and obſcure, it is here ſubjoined. *Ab oriente enim cum magno fulgore ſurgens, uſque ad medium hemiſphærii verſus occidentem, omnia perlucide pertrahebat.* ‘ In the year 1401 or 1402, the Sun being got below the horizon, there ap-

‘ peared in the weſt a bright and ſhin-
‘ ing Comet, ſending out a tail upwards
‘ in ſplendour like a flame of fire, and
‘ in form like a ſpear, darting its rays
‘ from weſt to eaſt. When the Sun
‘ was ſunk below the horizon, by the
‘ luſtre of its own rays it enlightned
‘ all the borders of the Earth, not per-
‘ mitting the other Stars to ſhew their
‘ light, or the ſhades of night to darken
‘ the air, becauſe its light exceeded that
‘ of the others, and extended it ſelf to
‘ the upper part of the heavens, fla-
‘ ming, *&c. Hiſt. Byzant. Duc. Mich. Nepot.* From the ſituation of the tail of this Comet, and the time of its firſt appearance, we may infer that the head was then near the Sun, and went farther from him every day. For that Comet continued three months. In the year 1527, *Aug.* 11, about four in the morning, there was ſeen almoſt throughout *Europe*, a terrible Comet in *Leo*, which continued flaming an hour and a quarter every day. It roſe from the eaſt, and aſcended to the ſouth and weſt to a prodigious length. It was moſt conſpicuous to the north, and its cloud (that is its tail) was very terrible; having, according to the fancies of the vulgar, the form of an arm a little bent, holding a ſword of a vaſt magnitude. In the

year 1618, in the end of *November*, there began a rumour, that there appeared about Sun-riſing a bright beam, which was the tail of a Comet, whoſe head was yet concealed within the brightneſs of the ſolar rays. On *Nov.* 24, and from that time, the Comet it ſelf appeared with a bright light, its head and tail being extremely reſplendent. The length of the tail, which was at firſt 20 or 30 deg. increaſed till *December* 9, when it aroſe to 75 deg. but with a light much more faint and dilute than at the beginning. In the year 1668, *March* 5. *N. S.* about ſeven in the evening P. *Valent. Eſtancius* being in *Braſile*, ſaw a Comet near the horizon in the ſouthweſt. Its head was ſmall, and ſcarce diſcernible, but its tail extremely bright and refulgent, ſo that the reflexion of it from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhore. This great ſplendour laſted but three days, decreaſing very remarkably from that time. The tail at the beginning extended it ſelf from weſt to ſouth, and in a ſituation almoſt parallel to the horizon, appearing like a ſhining beam 23 deg. in length. Afterwards the light decreaſing, its magnitude increaſed till the Comet ceaſed to be viſible. So that *Caſſini* at *Bologna* ſaw it (*Mar.* 10, 11, 12.) riſing from

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from the horizon 32 deg. in length. In *Portugal* it is ſaid to have taken up a fourth part of the heavens, (that is, 45 deg.) extending it ſelf from weſt to eaſt with a notable brightneſs; though the whole of it was not ſeen, becauſe the head in this part of the world always lay hid below the horizon. From the increaſe of the tail it is plain that the head receded from the Sun, and was neareſt to it at the beginning when the tail appeared brighteſt.

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To all theſe we may add the Comet of 1680, whoſe wonderful ſplendour at the conjunction of the head with the Sun was above deſcribed. But ſo great a ſplendour argues the Comets of this kind to have really paſſed near the fountain of light; eſpecially ſince the tails never ſhine ſo much in their oppoſition to the Sun; nor do we read that fiery beams have ever appeared there.

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The ſame proved from the light of their heads, as being greater *cæteris paribus* when they come near to the Sun.

LASTLY, the ſame thing is inferred from the light of the heads increaſing in the receſs of the Comets from the Earth towards the Sun, and decreaſing in their return from the Sun towards the Earth. For ſo the laſt Comet of the year 1665, (by the obſervation of *Hevelius*) from the time that it was firſt ſeen, was always loſing of its apparent motion, and therefore had already paſſed its perigee, yet the ſplendour of its head was daily increaſing, till being hid by the Sun's rays, the Comet ceaſed to appear. The Comet of the year 1683, (by the obſervation of the ſame *Hevelius*) about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orbit in a day's time. But from that time its diurnal motion was continually upon the increaſe till *September* 4, when it aroſe to about 5 degrees. And therefore in all this interval of time the Comet was approaching to the Earth. Which is likewiſe proved from the diameter of its head meaſured with a micrometer. For *Auguſt* the 6th *Hevelius* found it only 6′ 5″ including the coma; which *September* 2, he obſerved 9′ 7″. And therefore its head appeared far leſs about the beginning than towards the end of its motion, though about the beginning, becauſe nearer to the Sun, it appeared far more lucid than towards the end, as the ſame *Hevelius* declares. Wherefore in all this interval of time, on account of its receſs from the Sun, it decreaſed in ſplendour, notwithſtanding its acceſs towards the Earth. The Comet of the year 1618, about the middle of *December*, and that of the year 1680, about the end of the ſame month, did both move with their greateſt velocity, and were therefore then in their perigees. But the greateſt ſplendour of their heads was ſeen two weeks before when they had juſt got clear of the Sun's rays. And the greateſt ſplendour of their tails a little more early, when yet nearer to the Sun. The head of the former Comet, according to the obſervations of *Cyſatus*, *Dec.* 1, appeared greater than the Stars of the firſt magnitude; and *Dec.* 16, (being then in its perigee) of a ſmall magnitude, and the ſplendour or clearneſs was much diminiſhed. *Jan.* 7. *Kepler* being uncertain about the head, left off obſerving. *Dec.* 12. the head of the laſt Comet was ſeen and obſerved by *Flamſteed* at the diſtance of 9 degrees from the Sun; which a Star of the third magnitude could hardly have been. *December* 15 and 17, the ſame appeared like a Star of the third magnitude, its ſplendour being diminiſhed by the bright clouds near the ſetting Sun. *Dec.* 26, when it moved with the greateſt ſwiftneſs, and was almoſt in its perigee, it was inferiour to *Os Pegaſi*, a Star of the third magnitude. *Jan.* 3, it appeared like a Star of the fourth. *Jan.* 9, like a Star of the fifth. *Jan.* 13, it diſappeared by reaſon of the brightneſs of the Moon which was then in its increaſe. *Jan.* 25, it was ſcarce equal to the Stars of the ſeventh magnitude. If we take equal times on each hand of the perigee, the heads placed at remote diſtances would have ſhined equally before and after, becauſe of their equal diſtances from the Earth. That in one caſe they ſhined very bright, and in the other vaniſhed, is to be aſcribed to the nearneſs of the Sun in the firſt caſe, and his diſtance in the other. And from the great difference of the light in theſe two caſes, we infer its great nearneſs in the firſt of them. For the light of the Comets uſes to be regular, and to appear greateſt when their heads move the ſwifteſt, and are therefore in their perigees; excepting in ſo far as it is increaſed by their nearneſs to the Sun.

The ſame confirmed by the great number of Comets ſeen in the region of the Sun.

FROM theſe things I at laſt diſcovered why the Comets frequent ſo much the region of the Sun. If they were to be ſeen in the regions a great way beyond Saturn, they muſt appear oftner in thoſe parts of the heavens that are oppoſite to the Sun. For thoſe which are in that ſituation would be nearer to the Earth; and the interpoſition of the Sun would obſcure the others. But looking over the hiſtory of Comets, I find that four or five times more have been ſeen in the hemiſphere toward the Sun, than in the oppoſite hemiſphere; beſides, without doubt, not a few which have been hid by the light of the Sun. For Comets deſcending into our parts neither emit tails, nor are ſo well illuminated by the Sun as to diſcover themſelves to our naked eyes, till they are come nearer to us than Jupiter. But the far greater part of that ſphærical ſpace, which is deſcribed about the Sun with ſo ſmall an interval, lies on that ſide of the Earth which regards the Sun, and the Comets in that greater part are more ſtrongly illuminated, as being for the moſt part nearer to the Sun. Beſides, from the [*] remarkable eccentricity of their orbits it comes to paſs that their lower apſides are much nearer to the Sun than if their revolutions were performed in circles concentric to the Sun.

This alſo confirmed, by the greater magnitude and ſplendour of the tails after the conjunction of the heads with the Sun, than before.

HENCE alſo we underſtand why the tails of the Comets, while their heads are deſcending towards the Sun, always appear ſhort and rare, and are ſeldom ſaid to have exceeded 15 or 20 deg. in length; but in the receſs of the heads from the Sun often ſhine like fiery beams, and ſoon after reach to 40, 50, 60, 70 deg. in length, or more. This great ſplendour and length of the tails ariſes from the heat which the Sun communicates to the Comet as it paſſes near it. And thence I think it may be concluded that all the Comets that have had ſuch tails have paſſed very near the Sun.

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That the tails ariſe from the atmoſpheres of the Comets.

HENCE alſo we may collect that the tails ariſe from the atmoſpheres of the heads. But we have had three ſeveral opinions about the tails of Comets. For ſome will have it, that they are nothing elſe but the beams of the Sun's light tranſmitted through the Comets heads, which they ſuppoſe to be tranſparent: others, that they proceed from the refraction which light ſuffers in paſſing from the Comets head to the earth: and laſtly, others, that they are a ſort of clouds or vapour conſtantly riſing from the Comets heads, and tending towards the parts oppoſite to the Sun. The firſt is the opinion of ſuch as are yet unacquainted with opticks. For the beams of the Sun are not ſeen in a darkned room, but in conſequence of the light that is reflected from them, by the little particles of duſt and ſmoak which are always flying about in the air. And hence it is, that in air impregnated with thick ſmoak they appear with greater brightneſs, and are more faintly and more difficultly ſeen in a finer air. But in the heavens, where there is no matter to reflect the light, they are not to be ſeen at all. Light is not ſeen as it is in the beams, but as it is thence reflected to our eyes. For viſion is not made but by rays falling upon the eyes; and therefore there muſt be ſome reflecting matter in thoſe parts where the tails of Comets are ſeen; and ſo the argument turns upon the third opinion. For that reflecting matter can be no where found but in the place of the tail, becauſe otherwiſe, ſince all the celeſtial ſpaces are equally illuminated by the Sun's light, no part of the heavens could appear with more ſplendour than another. The ſecond opinion is liable to many difficulties. The tails of Comets are never ſeen variegated with thoſe colours, which ever uſe to be inſeparable from refraction. And the diſtinct tranſmiſſion of the light of the fixed Stars and Planets to us, is a demonſtration that the Æther or celeſtial medium is not endowed with any refractive power. For as to what is alledged that the fixed Stars have been ſometimes ſeen by the *Egyptians*, environed with a coma or capillitium, becauſe that has but rarely happened, it is rather to be aſcribed to a caſual refraction of clouds, as well as the radiation and ſcintillation of the fixed Stars to the refractions both of the eyes and air. For upon applying a teleſcope to the eye, thoſe radiations and ſcintillations immediately diſappear. By the tremulous agitation of the air and aſcending vapours, it happens that the rays of light are alternately turned aſide from the narrow ſpace of the pupil of the eye; but no ſuch thing can have place in the much wider aperture of the object-glaſs of a teleſcope. And hence it is, that a ſcintillation is occaſioned in the former caſe, which ceaſes in the latter. And this ceſſation in the latter caſe is a demonſtration of the regular tranſmiſſion of light through the heavens without any ſenſible refraction. But to obviate an objection that may be made from the appearing of no tail in ſuch Comets as ſhine but with a faint light, as if the ſecondary rays were then too weak to affect the eyes, and for this reaſon it is that the tails of the fixed Stars do not appear; we are to conſider, that by the means of teleſcopes the light of the fixed Stars may be augmented above an hundred fold, and yet no tails are ſeen. That the light of the Planets is yet more copious without any tail, but that Comets are ſeen ſometimes with huge tails, when the light of their heads is but faint and dull. For ſo it happened in the Comet of the year 1680, when in the month of *December* it was ſcarcely equal in light to the Stars of the ſecond magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60°, or 70°, and upwards; and afterwards on the 27th and 28th of *January*, the head appeared but as a Star of the 7th magnitude; but the tail (as was ſaid above) with a light that was ſenſible enough though faint, was ſtretched out to 6 or 7 degrees in length, and with a languiſhing light, that was more difficultly ſeen, even to 12° and upwards. But on the 9th and 10th of *February*, when to the naked eye the head appeared no more, I ſaw through a teleſcope, the tail of 2° in length. But further, if the tail was owing to the refraction of the celeſtial matter, and did deviate from the oppoſition of the Sun, according as the figure of the heavens requires; that deviation, in the ſame places of the heavens, ſhould be always directed towards the ſame parts. But the Comet

of the year 1680, *December* 28^{d} $8\frac{1}{2}^{h}$ P. M. at *London* was ſeen in Piſces 8° 41′ with latitude north 28° 6′, while the Sun was in Capricorn 18° 26′. And the Comet of the year 1577, *December* 29, was in Piſces 8° 41′, with latitude north 28° 40′, and the Sun as before in about Capricorn 18° 26′. In both caſes the ſituation of the Earth was the ſame, and the Comet appeared in the ſame place of the heavens; yet in the former caſe the tail of the Comet (as well by my obſervations as by the obſervations of others) deviated from the oppoſition of the Sun towards the north by an angle of 4½ degrees, whereas in the latter there was (according to the obſervation of *Tycho*) a deviation of 21 degrees towards the ſouth. The refraction therefore of the heavens being thus diſproved, it remains that the phænomena of the tails of Comets muſt be derived from
[*] ſome reflecting matter. That vapours ſufficient to fill ſuch immenſe ſpaces may ariſe from the Comets atmoſpheres, may be eaſily underſtood by what follows.

That the air and vapour in the celeſtial ſpaces is of an immenſe rarity: and that a ſmall quantity of vapour may be ſufficient to explain all the phænomena of the tails of Comets.

It is well known that the air near the ſurface of our Earth poſſeſſes a ſpace about 1200 times greater than water of the ſame weight. And therefore a cylindric column of air 1200 feet high, is of equal weight with a cylinder of water of the ſame breadth, and but one foot high. But a cylinder of air reaching to the top of the atmoſphere is of equal weight with a cylinder of water about 33 foot high; and therefore if from the whole cylinder of air the lower part of 1200 foot high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 foot high. Wherefore at the hight of 1200 feet, or two furlongs, the weight of the incumbent air is leſs, and conſequently the rarity of the compreſſed air greater than near the ſurface of the Earth in the ratio of 33 to 32. And having this ratio, we may compute the rarity of the air in all places whatſoever (by the help of *Cor. Prop.* 2. *Book* II.) ſuppoſing the expanſion thereof to be reciprocally proportional to its compreſſion; and this proportion has been proved by the experiments of *Hook* and others. The reſult of the computation I have ſet down in the following table, in the firſt column P. 366.

of which you have the hight of the air in miles, whereof 4000 make a ſemi-diameter of the Earth; in the ſecond the compreſſion of the air, or the incumbent weight; in the third its rarity or expanſion, ſuppoſing gravity to decreaſe in the duplicate ratio of the diſtances from the Earth's center. And the *Latin* numeral characters are here uſed for certain numbers of cyphers, as 0,xvii 1224 for 0,000000000000000001224, and 26956xv for 26956000000000000000.

AIR'S.

Hight.	Compreſſion.	Expanſion.
0	33	1
5	17,8515	1,8486
10	9,6717	3,4151
20	2,852	11,571
40	0,2525	136,83
400	0,xvii 1224	26956 xv
4000	0,cv 4465	73907 cii
40000	0,cxcii 1628	20263 clxxxix
400000	0,ccx 7895	41798 ccvii
4000000	0,ccxii 9878	33414 ccix
Infinite.	0,ccxii 6041	54622 ccix

But from this table it appears that the air, in proceeding upwards, is rarified in ſuch manner, that a ſphere of that air which is neareſt to the Earth, of but one inch in diameter, if dilated with that rarefaction which it would have at the hight of one ſemidiameter of the Earth, would fill all the planetary regions as far as the ſphere of Saturn, and a great way beyond; and at the hight of ten ſemidiameters of the Earth would fill up more ſpace than is contained in the whole heavens on this ſide the fixed Stars, according to the preceding computation of their diſtance. And though by reaſon of the far greater thickneſs of the atmoſpheres of Comets, and the great quantity of the circumſolar centripetal force, it may happen that the air in the celeſtial ſpaces, and in the tails of Comets, is not ſo vaſtly rarified; yet from this computation it is plain, that a very ſmall quantity of air and vapour is abundantly ſufficient to produce all the appearances of the tails of Comets. For that they are indeed of a very notable rarity appears from the ſhining of the Stars through them. The atmoſphere of the Earth, illuminated by the Sun's light, though but of a few miles in thickneſs, obſcures and extinguiſhes the light, not only of all the Stars, but even of the Moon itſelf; whereas the ſmalleſt Stars are ſeen to ſhine through the immenſe thickneſs of the tails of Comets, likewiſe illuminated by the Sun, without the leaſt diminution of their ſplendor. [*]

P. 369.

After what manner the tails of Comets may ariſe from the atmoſpheres of their heads.

KEPLER aſcribes the aſcent of the tails of Comets to the atmoſpheres of their heads, and their direction towards the parts oppoſite to the Sun, to the action of the rays of light carrying along with them the matter of the Comets tails. And without any great incongruity we may ſuppoſe that, in ſo free ſpaces, ſo fine a matter as that of the Æther may yield to the action of the rays of the Sun's light, though thoſe rays are not able ſenſibly to move the groſs ſubſtances in our parts, which are clogged with ſo palpable a reſiſtance. Another author thinks that there may be a ſort of particles of matter endowed with a principle of levity as well as others are with a power of gravity; that the matter of the tails of Comets may be of the former ſort, and that its aſcent from the Sun may be owing to its levity. But conſidering the gravity of terreſtrial bodies is as the matter of the bodies, and therefore can be neither more nor leſs in the ſame quantity of matter, I am inclined to believe that this aſcent may rather proceed from the rarefaction of the matter of the Comets tails. The aſcent of ſmoak in a chimney is owing to the impulſe of the air, with which it is intangled. The air rarified by heat aſcends, becauſe its ſpecifick gravity is diminiſhed, and in its aſcent carries along with it the ſmoak with which it is ingaged. And why may not the tail of a Comet riſe from the Sun after the ſame manner? For the Sun's rays do not act any way upon the mediums which they pervade, but by reflection and refraction. And thoſe reflecting particles heated by this action, heat the matter of the Æther which is involved with them. That matter is rarefied by the heat which it acquires, and becauſe by this rarefaction the ſpecifick gravity, with which it tended towards the Sun before, is diminiſhed, it will aſcend therefrom like a ſtream, and carry along with it the reflecting particles of which the tail of the Comet is compoſed; the impulſe [* of the Sun's light, as we have ſaid, promoting the aſcent.

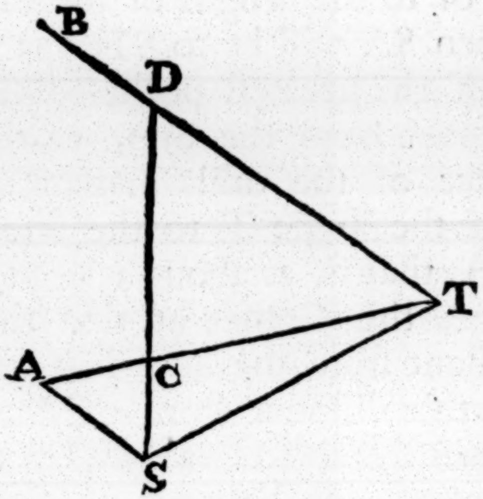
That the tails do indeed ariſe from thoſe atmoſpheres, proved from ſeveral of their phænomena.

P. 364.

But that the tails of Comets do ariſe from their heads and tend towards the parts oppoſite to the Sun, is further confirmed from the laws which the tails obſerve. For lying in the plains of the Comets orbits which paſs through the Sun, they conſtantly deviate from the oppoſition of the Sun, towards the parts which the Comets heads in their progreſs along thoſe orbits have left. And to a ſpectator placed in thoſe plains, they appear in the parts directly oppoſite to the Sun. But as the ſpectator recedes from thoſe plains, their deviation begins to appear, and daily becomes greater. And the deviation *cæteris paribus* appears leſs, when the tail is more oblique to the orbit of the Comet, as well as when the head of the Comet approaches nearer to the Sun; eſpecially if the angle of deviation is eſtimated near the head of the Comet. Farther, the tails which have no deviation appear ſtreight, but the tails which deviate are likewiſe bended into a certain curvature. And this curvature is greater when the deviation is greater; and is more ſenſible when the tail *cæteris paribus* is longer: for in the ſhorter tails the curvature is hardly to be perceived. And the angle of deviation is leſs near the Comet's head, but greater towards the other end of the tail, and that becauſe the lower ſide of the tail regards the parts from which the deviation is made, and which lie in a right line, drawn out infinitely from the Sun through the Comet's head. And the tails that are longer and broader, and ſhine with a ſtronger light, appear more reſplendent and more exactly defined on the convex than on the concave ſide. Upon which accounts it is plain that the phænomena of the tails of Comets depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen; and that therefore the tails of the Comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail. For as in our air the ſmoak of a heated body aſcends either perpendicularly, if the body is at reſt, or obliquely, if the body is moved obliquely; ſo in the heavens, where all the bodies gravitate towards the Sun, ſmoak and vapour muſt (as we have already ſaid) aſcend from the Sun, and either riſe perpendicularly if the ſmoaking body is at reſt, or obliquely if the body, in the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapours had riſen before. And that obliquity will be leſs, where the vapour aſcends with more velocity, to wit, near the ſmoaking body when that is near the Sun. For there the force of the Sun by which the vapour aſcends is ſtronger. But becauſe the obliquity is varied, the column of vapour will be incurvated; and becauſe the vapour in the preceding ſide is ſomething more recent, that is, has aſcended ſomething more lately from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light as well as be better defined; the vapour on the other ſide languiſhing by degrees and vaniſhing out of ſight. [*]

That Comets do ſometimes deſcend below the orbit of Mercury, proved from their tails.

But it is none of our preſent buſineſs to explain the cauſes of the appearances of nature. Let thoſe things which we have laſt ſaid be true or falſe; we have at leaſt made out in the preceding diſcourſe, that the rays of light are directly propagated from the tails of Comets in right lines through the heavens, in which thoſe tails appear to the ſpectators where-ever placed; and conſequently the tails muſt aſcend from the heads of the Comets towards the parts oppoſite to the Sun. And from this principle we may determine anew the limits of their diſtances in manner following. Let S repreſent the Sun, T the Earth, STA the elongation of a Comet from the Sun, and ATB the apparent length of its tail. And becauſe the light is propagated from the extremity of the tail in the direction of the right line TB, that extremity muſt lie ſomewhere in the line TB. Suppoſe it in D, and join DS cutting TA in C. Then becauſe the tail is always ſtretched out towards the parts nearly oppoſite to the Sun; and therefore the Sun, the head of the Comet, and the extremity of the tail lie in a right line, the Comet's head will be found in C. Parallel to TB draw SA, meeting the line TA in A, and the Comet's head C muſt neceſſarily be found between T and A, becauſe the extremity of the tail lies ſomewhere in the infinite line TB; and all the lines SD which can poſſibly be drawn from the point S to the line TB, muſt cut the line TA ſomewhere between T and A.



Wherefore the diſtance of the Comet from the Earth cannot exceed the interval TA, nor its diſtance from the Sun the interval SA beyond, or ST on this ſide the Sun. For inſtance, the elongation of the Comet of 1680 from the Sun *Dec.* 12. was 9° and the length of its tail 35° at leaſt. If therefore a triangle TSA is made, whoſe angle T is equal to the elongation 9° and angle A equal to ATB, or to the length of the tail, *viz.* 35° then SA will be to ST, that is, the limit of the greateſt poſſible diſtance of the Comet from the Sun, to the ſemidiameter of the orbis magnus, as the ſine of the angle T to the ſine of the angle A, that is, as about 3 to 11. And therefore the Comet at that time was leſs diſtant from the Sun than by $\frac{3}{11}$ of the Earth's diſtance from the Sun, and conſequently either was within the orb of Mercury, or between that orb and the Earth. Again, *Dec.* 21. the elongation of the Comet from the Sun was $32\frac{2}{3}^{\circ}$, and the length of its tail 70°. Wherefore as the ſine of $32^{\circ}\frac{2}{3}^{\circ}$ to the ſine of 70°, that is, as 4 to 7, ſo was the limit of the Comet's diſtance from the Sun to the diſtance of the Earth from the Sun, and conſequently the Comet had not then got without the orb of

Venus. *Dec.* 28. the elongation of the Comet from the Sun was 55° and the length of its tail 56°. And therefore the limit of the Comet's diſtance from the Sun was not yet equal to the diſtance of the Earth from the ſame, and conſequently the Comet had not then got without the Earth's orbit. But from its parallax we find that its egreſs from the orbit happened about *Jan.* 5, as well as that it had deſcended far within the orbit of Mercury. Let us ſuppoſe it to have been in its perihelion *Dec.* the 8th when it was in conjunction with the Sun, and it will follow that in the journey from its perihelion to its exit out of the Earth's orbit, it had ſpent 28 days; and conſequently that in the 26 or 27 days following, in which it ceaſed to be further ſeen by the naked eye, it had ſcarce doubled its diſtance from the Sun. And by limiting the diſtances of other Comets by the like arguments, we come at laſt to this concluſion, That all Comets during the time in which they are viſible by us, are within the compaſs of a ſpherical ſpace deſcribed about the Sun as a center, with a radius double, or at moſt triple of the diſtance of the Earth from the Sun.

That the Comets move in conic ſections, having one focus in the center of the Sun, and by radii drawn to that center do deſcribe area's proportional to the times.

AND hence it follows that the Comets, during the whole time of their appearance unto us, being within the ſphere of activity of the circum-ſolar force, and therefore agitated by the impulſe of that force, will (by *Cor.* 1. *Prop.* 13. for the ſame reaſon as the Planets) be made to move in conic ſections that have one focus in the center of the Sun, and, by radii drawn to the Sun, to deſcribe area's proportional to the times. [✱] For that force is propagated to an immenſe diſtance, and will govern the motions of bodies far beyond the orbit of Saturn.

P. 332.

That thoſe conic ſections are near to parabola's, proved from the velocity of the Comets.

THERE are three hypotheſes about Comets. For ſome will have it that they are generated and periſh, as often as they appear and vaniſh; others, that they come from the regions of the fixed Stars, and are ſeen by us in their paſſage through the ſyſtem of our Planets; and laſtly, others that they are bodies perpetually revolving about the Sun in very eccentric orbits. In the firſt caſe, the Comets, according to their different velocities, will move in conic ſections of all ſorts; in the ſecond they will deſcribe hyperbola's, and in either of the two will frequent indifferently all quarters of the heavens as well thoſe about the poles as thoſe towards the Ecliptic; in the third their motions will be performed in ellipſes very eccentric and very nearly approaching to parabola's. But (if the law of the Planets is obſerved) their orbits will not much decline from the plain of the Ecliptic. And ſo far as I could hitherto obſerve the third caſe obtains. For the Comets do indeed chiefly frequent the zodiac and ſcarce ever attain to a heliocentric latitude of 40°. And that they move in orbits very nearly parabolical, I infer from their velocity. For the velocity with which a parabola is deſcribed, is every where to the velocity with which a Comet or Planet may be revolved about the Sun in a circle at the ſame diſtance, in the ſubduplicate ratio of 2 to 1. (by *Cor.* 7. *Prop.* 16.) And by my computation the velocity of Comets is found to be much about the ſame. I examined the thing by inferring nearly the velocities from the diſtances, and the diſtances both from the parallaxes and the phænomena of the tails, and never found the errours of exceſs or defect in the velocities greater than what might have aroſe from the errours in the diſtances collected after that manner. But I likewiſe made uſe of the reaſoning that follows.

In what ſpace of time Comets deſcribing parabolic trajectories paſs through the ſphere of the orbis magnus.

SUPPOSING the radius of the orbis magnus to be divided into 1000 parts: Let the numbers in the firſt column of the following table repreſent the diſtance of the vertex of the parabola from the Sun's center, expreſſed by thoſe parts; and a Comet in the times expreſſed in Col. 2. will paſs from its perihelion to the ſurface of the ſphere which is deſcribed about the Sun as a center with the radius of the orbis magnus; and in the times expreſſed in Col. 3, 4, and 5, it will double, triple, and quadruple that its diſtance from the Sun.

TABLE I.

The diſtance of a Comet's perihelion from the Sun's center.	The time of a Comet's paſſage from its perihelion to a diſtance from the Sun equal to											
	The rad. of the orbis mag.			To its double.			To its triple.			To its quadruple.		
	d.	h.	′.	d.	h.	′.	d.	h.	′.	d.	h.	′.
0	27	11	12	77	16	28	142	17	14	219	17	30
5	27	16	07	77	23	14						
10	27	21	00	78	06	24						
20	28	06	40	78	20	13	144	03	19	221	08	54
40	29	01	32	79	23	34						
80	30	13	25	82	04	56						
160	33	05	29	86	10	26	153	16	08	232	12	20
320	37	13	46	93	23	38						
640	37	09	49	105	01	28						
1280				106	06	35	200	06	43	297	03	46
2560							147	22	31	300	06	03

[This table, here corrected, is made on the ſuppoſition that the Earth's diurnal motion is juſt 59′, and the meaſure of one minute looſely 0,2909, in reſpect of the radius 1000. If thoſe meaſures are taken true, the true numbers of the table will all come out leſs. But the difference, even when greateſt, and to the quadruple of the Earth's diſtance from the Sun, amounts only to 16^{h}. 55′.]

At what time Comets enter into and paſs out of the ſphere of the orbis magnus.

THE time of a Comet's ingreſs into the ſphere of the orbis magnus, or of its egreſs from the ſame may be inferred nearly from its parallax, but with more expedition by the following

TABLE II.

The apparent elongation of a Comet from the Sun.	Its apparent diurnal motion in its own orbit.		Its diſtance from the Earth in parts, whereof the radius of the orbis magnus contains 1000.
	Direct.	Retrog.	
60°	2° 18′	00° 20′	1000
65	2 33	00 35	845
70	2 55	00 57	684
72	3 07	01 09	618
74	3 23	01 25	551
76	3 43	01 45	484
78	4 10	02 12	416
80	4 57	02 49	347
82	5 45	03 47	278
84	7 18	05 20	209
86	10 27	08 19	140
88	18 37	16 39	70
90	Infinite	Infinite	00

With what velocity the Comets of 1680 paſſed through the ſphere of the orbis magnus.

THE ingreſs of a Comet into the ſphere of the orbis magnus, or its egreſs from the ſame happens at the time of its elongation from the Sun, expreſſed in Col. 1. againſt its diurnal motion.

So in the Comet of 1681, *Jan.* 4. *O. S.* the apparent diurnal motion in its orbit was about $3^{\circ}\ 5'$ and the correſponding elongation $71\frac{2}{3}^{\circ}$. And the Comet had acquired this elongation from the Sun *Jan.* 4, about ſix in the evening. Again, in the year 1680, *November* 11, the diurnal motion of the Comet that then appeared, was about $4\frac{2}{3}^{\circ}$, and the correſponding elongation $79\frac{2}{3}$ happened *Nov.* 10, a little before mid-night. Now at the times named theſe Comets had arrived at an equal diſtance from the Sun with the Earth, and the Earth was then almoſt in its perihelion. But the firſt table is fitted to the Earth's mean diſtance from the Sun aſſumed of 1000 parts; and this diſtance is greater by ſuch an exceſs of ſpace as the Earth might deſcribe, by its annual motion, in one day's time, or the Comet, by its motion, in 16 hours. To reduce the Comet to this mean diſtance of 1000 parts we add thoſe 16 hours to the former time, and ſubduct them from the latter; and thus the former becomes *Jan.* 4^{d} 10^{h} after noon, the latter *Nov.* 10, about ſix in the morning. But from the tenour and progreſs of the diurnal motions it appears that both Comets were in conjunction with the Sun between *Dec.* 7. and *Dec.* 8. And from thence to *Jan.* 4^{d} 10^{h} afternoon on one ſide, and to *Nov.* 10^{d} 6^{h} of the morning on the other, there are about 28 days. And ſo many days (by *Table* 1.) the motions in parabolic trajectories do require.

That theſe were not two, but one and the ſame Comet. In what orbit and with what velocity this Comet was carried through the heavens deſcribed more exactly.

BUT though we have hitherto conſidered thoſe Comets as two, yet from the coincidence of their perihelions and agreement of their velocities, it is probable, that in effect they were but one and the ſame. And if ſo, the orbit of this Comet muſt have either been a parabola or at leaſt a conic ſection very little differing from a parabola, and at its vertex almoſt in contact with the ſurface of the Sun. For by *Tab.* 2. the diſtance of the Comet from the Earth *Nov.* 10, was about 360 parts, and *Jan.* 4, about 630. From which diſtances, together with its longitudes and latitudes, we infer the diſtance of the places in which the Comet was at thoſe times, to have been about 280: the half of which, *viz.* 140 is an ordinate to the Comet's orbit, cutting off a portion of its axe nearly equal to the radius of the orbis magnus, that is to 1000 parts. And therefore dividing the ſquare of the ordinate 140 by 1000 the ſegment of the axe, we find the latus rectum 19,16, or in a round number 20; the fourth part whereof 5 is the diſtance of the vertex of the orbit from the Sun's center. But the time correſponding to the diſtance of 5 parts in *Tab.* 1. is 27^{d} 16^{h} $7'$. In which time, if the Comet moved in a parabolic orbit, it would have been carried from its perihelion to the ſurface of the ſphere of the orbis magnus deſcribed with the radius 1000, and would have ſpent the double of that time, *viz.* 55^{d} $8\frac{1}{4}^{h}$ in the whole courſe of its motion within that ſphere: And ſo in fact it did. For from *Nov.* 10^{d} 6^{h} of the morning, the time of the Comet's ingreſs into the ſphere of the orbis magnus, to *Jan.* 4^{d} 10^{h} after noon, the time of its egreſs from the ſame, there are 55^{d} 16^{h}. The ſmall difference of $7\frac{3}{4}^{h}$ in this rude way of computing is to be neglected, and perhaps may ariſe from the Comet's motion being ſome ſmall matter ſlower, as it muſt have been if the true orbit in which it was carried was an ellipſe. The middle time between its ingreſs and egreſs was *December* 8^{d} 2^{h} of the morning. And therefore at this time the Comet ought to have been in its perihelion. And accordingly that very day, just before Sun-riſing, Dr. *Halley* (as we ſaid) ſaw the tail ſhort and broad, but very bright, riſing perpendicularly from the horizon. From the poſition of the tail, it is certain that the Comet had then croſſed over the Ecliptic, and got into north-latitude, and therefore had paſſed by its perihelion which lay on the other ſide of the Ecliptic, though it had not yet come into conjunction with the Sun. And the Comet, being at this time between its perihelion and its conjunction with the Sun, muſt have been in its perihelion a few hours before. For in ſo near a diſtance from the Sun it muſt have been carried with great velocity, and have apparently deſcribed almoſt half a degree every hour.

See more of this famous Comet from Pag. 344 *to P.* 361. Vol. II. Math. Princip.

With what velocity Comets are carried, ſhewed by more examples.

By like computations I find that the Comet of 1618 entered the ſphere of the orbis magnus *December* 7, towards Sun-ſetting. But its Conjunction with the Sun was *Nov.* 9, or 10, about 28 days intervening, as in the preceding Comet. For from the ſize of the tail of this, in which it was equal to the preceding, it is probable that this Comet likewiſe did come almoſt into a contact with the Sun. Four Comets were ſeen that year, of which this was the laſt. The ſecond, which made its firſt appearance *October*

ber 31, in the neighbourhood of the riſing Sun, and was ſoon after hid under the Sun's rays, I ſuſpect to have been the ſame with the fourth, which emerged out of the Sun's rays about *Nov.* 9. To theſe we may add the Comet of 1607, which entered the ſphere of the orbis magnus *Sept.* 14. *O S.* and arrived at its perihelion-diſtance from the Sun about *October* 19, 35 days intervening. Its perihelion-diſtance ſubtended an apparent angle at the Earth of about 23 degrees, and was therefore of 390 parts. And to this number of parts about 34 days correſpond in *Tab.* 1. Further, the Comet of 1665 entered the ſphere of the orbis magnus about *March* 17, and came to its perihelion about *April* 16, 30 days intervening. Its perihelion-diſtance ſubtended an angle at the Earth of about ſeven degrees, and therefore was of 122 parts: and correſponding to this number of parts, in *Tab.* 1. we find 30 days. Again, the Comet of 1682 entered the ſphere of the orbis magnus about *Aug.* 11, and arrived at its perihelion about *Sept.* 16, being then diſtant from the Sun by about 350 parts, to which, in *Tab.* 1. belong 33½ days. Laſtly, that memorable Comet of *Regiomontanus*, which in 1472 was carried through the circum-polar parts of our northern hemiſphere with ſuch rapidity as to deſcribe 40 degrees in one day, entered the ſphere of the orbis magnus *Jan* 21. about the time that it was paſſing by the pole, and haſtening from thence towards the Sun, was hid under the Sun's rays about the end of *February*. Whence 'tis probable that 30 days or a few more were ſpent between its ingreſs into the ſphere of the orbis magnus and its perihelion. Nor did this Comet truly move with more velocity than other Comets, but owed the greatneſs of its apparent velocity to its paſſing by the Earth at a near diſtance.

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The inveſtigation of the trajectory of Comets propoſed.

It appears then that the velocity of Comets, ſo far as it can be determined by theſe rude ways of computing, is that very velocity with which parabola's, or ellipſes near to parabola's, ought to be deſcribed. And therefore the diſtance between a Comet and the Sun being given, the velocity of the Comet is nearly given. And hence ariſes this problem.

PRO-

PROBLEM.

The relation betwixt the velocity of a Comet, and its diſtance from the Sun's center being given, the Comet's trajectory is required.

IF this problem was reſolved, we ſhould thence have a method of determining the trajectories of Comets to the greateſt accuracy. For if that relation be twice aſſumed, and from thence the trajectory be twice computed, and the errour of each trajectory be found from obſervations, the aſſumption may be corrected by the Rule of Falſe, and a third trajectory may thence be found that will exactly agree with the obſervations. And by determining the trajectories of Comets after this method, we may come at laſt to a more exact knowledge of the parts through which thoſe bodies travel, of the velocities with which they are carried, what ſort of trajectories they deſcribe, and what are the true magnitudes and forms of their tails according to the various diſtances of their heads from the Sun;

whether after certain intervals of time, the ſame Comets do return again, and in what periods they compleat their ſeveral revolutions. But the problem may be reſolved by determining firſt the hourly motion of a Comet to a given time from three or more obſervations, and then deriving the trajectory from this motion. And thus the invention of the trajectory, depending on one obſervation and its hourly motion at the time of this obſervation, will either confirm or diſprove it ſelf. For the concluſion that is drawn from the motion only of an hour or two and a falſe hypotheſis, will never agree with the motions of the Comets from beginning to end. The method of the whole computation is this.

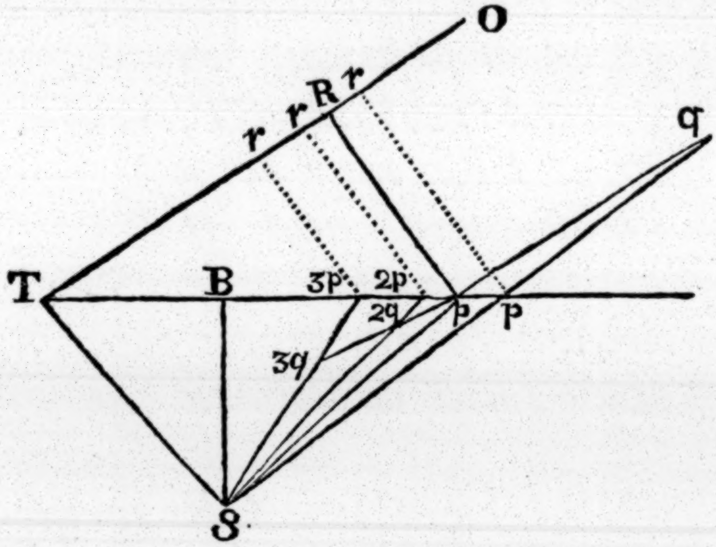
LEM-

LEMMA I.

To cut two right lines OR, TP given in poſition, by a third right line RP, ſo as TRP may be a right angle, and, if another right line SP is drawn to any given point S, the ſolid contained under this line SP and the ſquare of the right line OR terminated at a given point O, may be of a given magnitude.

Lemma's premiſed to the ſolution of the problem.

It is done by linear deſcription, thus. Let the given magnitude of the ſolid be $M^2 \times N$. From any point *r* of the right line OR erect the perpendicular *rp* meeting TP in *p*. Then through the points S*p* draw the line S*q* equal to $\frac{M^2 \times N}{Or^2}$. In like manner draw three or more right lines S2*q*, S3*q*, &c. And a regular line *q*2*q*3*q* drawn through all the points *q*2*q*3*q*, &c. will cut the right line TP in the point P, from which the perpendicular PR is to be let fall. Q. E. F.



By trigonometry thus. Aſſuming the right line TP as found by the preceding method, the perpendiculars TR, SB in the triangles TPR, TPS will be thence given, and the ſide SP in the triangle SBP, as well as the errour $\frac{M^2 \times N}{OR^2} - SP$. Let this errour, ſuppoſe D, be to a new errour, ſuppoſe E, as the errour $2p2q \pm 3p3q$ to the errour $2p3p$; or as the errour $2p2q \mp D$ to the errour $2pP$; and this new errour added to or ſubducted from the length TP will give the correct length TP ± E. The inſpection of the figure will ſhew whether we are to add or to ſubtract. And if at any time there ſhould be uſe for a further correction, the operation may be repeated.

By arithmetic thus. Let us ſuppoſe the thing done, and let TP $+ e$ be the correct length of the right line TP as found out by delineation; and thence the correct lengths of the lines OR, BP and SP will be $OR - \frac{TR}{TP}e$, $BP + e$, and $\sqrt{SP^2 + 2\,BPe + ee}$

$$= \frac{M^2 N}{OR^2 + \frac{2OR + TR}{TP} e + \frac{TR^2}{TP^2} ee}.$$

Whence by the method of converging ſeries, we have $SP + \frac{BP}{SP} e + \frac{SB^2}{2SP^3} ee$, *&c.*

$$= \frac{M^2 N}{OR^2} + \frac{2\,TR}{TP} \times \frac{M^2 N}{OR^3} e + \frac{3TR^2}{TP^2} \times \frac{M^2 N}{OR^4} ee, \text{ \&c.}$$

For the given co-efficients $\frac{M^2 N}{OR^2} - SP$, $\frac{2\,TR}{TP} \times \frac{M^2 N}{OR^3} - \frac{BP}{SP}$, $\frac{3TR^2}{TP^2} \times \frac{M^2 N}{OR^4} - \frac{SB^2}{2SP^3}$, putting F, $\frac{F}{G}$, $\frac{F}{GH}$, and carefully obſerving the ſigns, we find $F + \frac{F}{G} e + \frac{F}{GH} ee = 0$, and $e + \frac{ee}{H} = -G$.

Whence, neglecting the very ſmall term $\frac{e^2}{H}$, e comes out equal to $-G$.

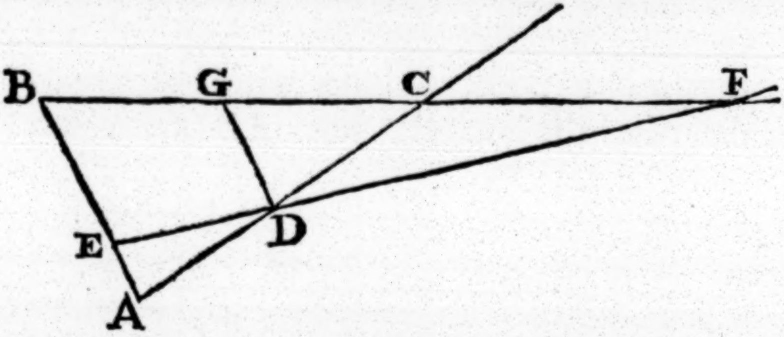
If the errour $\frac{e^2}{H}$, is not deſpicable, take $-G-\frac{G^2}{H}=e$.

And it is to be obſerved, that here a general method is hinted at for ſolving the more intricate ſort of problems, as well by trigonometry as by arithmetic, without thoſe perplexed computations and reſolutions of affected equations, which hitherto have been in uſe.

LEMMA II.

To cut three right lines given in poſition by a fourth right line that ſhall paſs through a point aſſigned in any of the three, and ſo as its intercepted parts ſhall be in a given ratio one to the other.

Let AB, AC, BC be the right lines given in poſition, and ſuppoſe D to be the given point in the line AC. Parallel to AB draw DG meeting BC in G. And,



taking GF to BG in the given ratio, draw FDE; and FD will be to DE as FG to BG. Q. E. F.

By trigonometry thus. In the triangle CGD, all the angles and the ſide CD are given, and from thence its remaining ſides are found; and from the given ratio's, the lines GF and BE are alſo given.

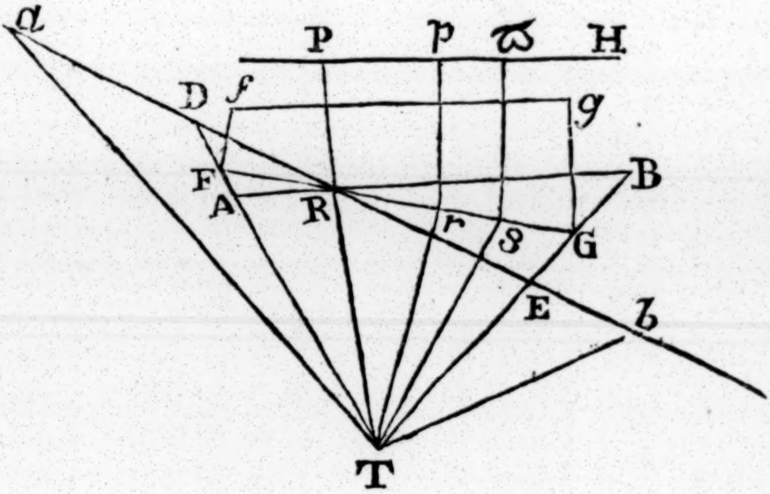
LEMMA III.

To find and repreſent by a linear deſcription the hourly motion of a Comet to any given time.

From obſervations of the beſt credit, let three longitudes of the Comet be given, and ſuppoſing ATR, RTB to be their differences; let the hourly motion be required to the time of the middle

obſer-

obſervation TR. By *Lem.* II. draw the right line ARB, ſo as its intercepted parts AR, RB may be as the times between the obſervations. And if we ſuppoſe a body in the whole time to deſcribe the whole line AB with an



equal motion, and to be in the mean time viewed from the place T, the apparent motion of that body about the point R, will be nearly the ſame with that of the Comet at the time of the obſervation TR.

The ſame more accurately.

Let T*a*, T*b* be two longitudes given at a greater diſtance on one ſide and on the other; and by *Lem.* II. draw the right line *a*R*b* ſo as its intercepted parts *a*R, R*b* may be as the times between the obſervations *a*TR, RT*b*. Suppoſe this to cut the lines TA, TB in D and E. And becauſe the errour of the inclination TR*a* increaſes nearly in the duplicate ratio of the time between the obſervations, draw FRG, ſo as either the angle DRF may be to the angle ARF, or the line DF to the line AF, in the duplicate ratio of the whole time between the obſervations *a*TB to the whole time between the obſervations ATB, and uſe the line thus found FG in place of the line AB found above.

It will be convenient that the angles ATR, RTB, *a*TA, BT*b* be no leſs than of ten or fifteen degrees, the times correſponding no greater than of eight or twelve days, and the longitudes taken when the Comet moves with the greateſt velocity. For thus the errours of the obſervations will bear a leſs proportion to the differences of the longitudes.

LEMMA IV.

To find the longitudes of a Comet to any given times.

It is done by taking, in the line FG, the diſtances R*r*, Rρ proportional to the times, and drawing the lines T*r*, Tρ. The way of working by trigonometry is manifeſt.

LEMMA V.

To find the latitudes.

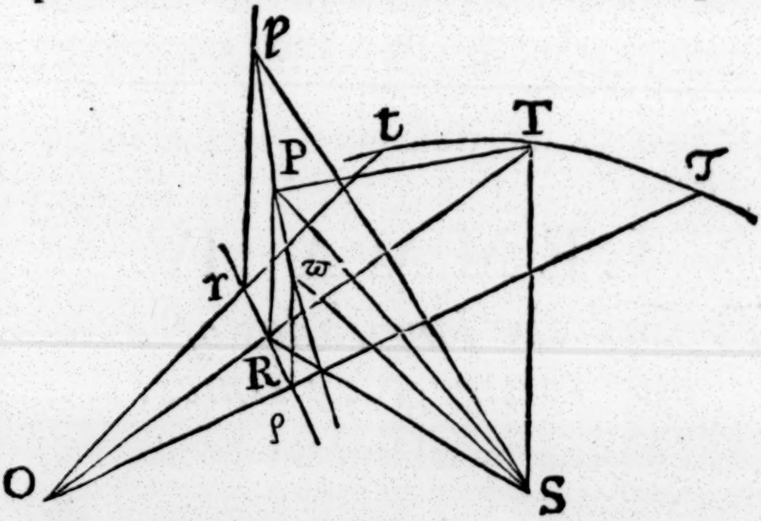
On TF, TR, TG, as radius's, at right angles erect F*f*, RP, G*g* tangents of the obſerved latitudes; and parallel to *fg* draw PH. The perpendiculars *rp*, *ρϖ*, meeting PH, will be the tangents of the ſought latitudes to T*r* and T*ρ* as radius's.

PROBLEM. I.

From the aſſumed ratio of the velocity to determine the trajectory of a Comet.

The problem reſolved.

Let S repreſent the Sun, *t*,T,*τ* three places of the Earth in its orbit at equal



diſtances, *p*,P,*ϖ* as many correſponding places of the Comet in its trajectory, ſo as the diſtances interpoſed betwixt place and place may anſwer to the motion of one hour; pr, PR, $\varpi\rho$, perpendiculars let fall on the plain of the ecliptic, and $rR\varrho$ the veſtige of the trajectory in this plain. Join Sp, SP, $S\varpi$, SR, ST, tr, TR, $\tau\rho$, TP; and let tr, $\tau\rho$ meet in O, TR will nearly converge to the ſame point O, or the errour will be inconſiderable. By the premiſed lemma's, the angles rOR, $RO\rho$ are given, as well as the ratio's pr to tr, PR to TR, and $\varpi\rho$ to $\tau\rho$. The figure $tT\tau O$ is likewiſe given, both in magnitude and poſition, together with the diſtance ST and the angles STR, PTR, STP. Let us aſſume the velocity of the Comet, in the place P, to be to the velocity of a Planet, revolved about the Sun in a circle, at the ſame diſtance SP, as V to 1, and we ſhall have a line $pP\varpi$ to be determined, of this condition, that the ſpace $p\varpi$, deſcribed by the Comet in two hours, may be to the ſpace $V \times t\tau$ that is, to the ſpace which the Earth deſcribes in the ſame time multiplied by the number V) in the ſubduplicate ratio of ST, the diſtance of the Earth from the Sun, to SP the diſtance of the Comet from the Sun; and that the ſpace pP, deſcribed by the Comet in the firſt hour, may be to the ſpace $P\varpi$ deſcribed by the Comet in the ſecond hour, as the velocity in p, to the velocity in P, that is, in the ſubduplicate ratio of the diſtance SP to the diſtance Sp, or in the ratio of $2Sp$, to $SP + Sp$. For in this whole work I neglect ſmall fractions that can produce no ſenſible errour.

In the firſt place then, as mathematicians in the reſolution of affected equations, are wont, for the firſt eſſay, to aſſume the root by conjecture, ſo, in this analytical operation, I judge of the ſought diſtance TR, as I beſt can by conjecture. Then by *Lem.* II. I draw $r\rho$, firſt ſuppoſing rR equal to $R\rho$, and again (after the ratio of SP to Sp is diſcovered) ſo as rR may be to $R\rho$, as $2SP$ to $SP + Sp$, and I find the ratio's of the lines $p\varpi$, $r\rho$ and OR one to the other. Let M be to $V \times t\tau$ as OR to $p\varpi$; and becauſe the ſquare of $p\varpi$ is to the ſquare of $V \times t\tau$, as ST to SP, we ſhall have *ex æquo* OR^2 to M^2 as ST to SP, and therefore the ſolid $OR^2 \times SP$ equal to the given ſolid $M^2 \times ST$. Whence (ſuppoſing the triangles STP, PTR to be now placed in the ſame plain) TR, TP, SP, PR will be given by *Lem.* I. All this I do, firſt by delineation in a rude and haſty way, then by a new delineation with greater care, and laſtly, by an arithmetical computation. Then I proceed to determine the poſition of the lines rp, $p\varpi$ with the greateſt accuracy, together with the nodes and inclination of the plain $Sp\varpi$ to the plain of the ecliptic; and in that plain $Sp\varpi$, I deſcribe the trajectory in which a body let go from the place P in the direction of the given right line $p\varpi$, would be carried with a velocity that is to the velocity of the Earth, as $p\varpi$ to $V \times t\tau$. Q.E.F.

PROBLEM II.

To correct the aſſumed ratio of the velocity and the trajectory thence found.

Take an obſervation of the Comet about the end of its appearance, or any other obſervation at a very great diſtance from the obſervations uſed before, and find the interſection of a right line drawn to the Comet, in that obſervation, with the plain $Sp\varpi$, as well as the Comet's place in its trajectory to the time of the obſervation. If that interſection happens in this place, it is a proof that the trajectory was right determined. If otherwiſe, a new number V is to be aſſumed, and a new trajectory to be found, and then the place of the Comet in this trajectory to the time of that probatory obſervation, and the interſection, of a right line drawn to the Comet, with the plain of the trajectory, are to be determined as before. And by comparing the variation of the errour with the variation of the other quantities, we may conclude, by the Rule of Three, how far thoſe other quantities ought to be varied or corrected, ſo as the errour may become as ſmall as poſſible. And by means of theſe corrections we may have the trajectory exactly, providing the obſervations, upon which the computation was founded, were exact, and that we did not err much in the aſſumption of the quantity V; for if we did, the operation is to be repeated till the trajectory is exactly enough determined. Q. E. F.

FINIS.

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